

## ***IX. VOLUMES AND AREAS<sup>114</sup>***

The particle flux in Monte Carlo transport problems often is estimated as the track length per unit volume or the number of particles crossing a surface per unit area. Therefore, knowing the volumes and surface areas of the geometric regions in a Monte Carlo problem is essential. Knowing volumes is useful in calculating the masses and densities of cells and thus in calculating volumetric or mass heating. Furthermore, calculation of the mass of a geometry is frequently a good check on the accuracy of the geometry setup when the mass is known by other means.

Calculating volumes and surface areas in modern Monte Carlo transport codes is nontrivial. MCNP allows the construction of cells from unions and/or intersections of regions defined by an arbitrary combination of second-degree surfaces, toroidal fourth-degree surfaces, or both. These surfaces can have different orientations or be segmented for tallying purposes. The cells they form even can consist of several disjoint subcells. Cells can be constructed from quadrilateral or hexagonal lattices or can be embedded in repeated structures universes. Although such generality greatly increases the flexibility of MCNP, computing cell volumes and surface areas understandably requires increasingly elaborate computational methods.

MCNP automatically calculates volumes and areas of polyhedral cells and of cells or surfaces generated by surfaces of revolution about any axis, even a skew axis. If a tally is segmented, the segment volumes or areas are computed. For nonrotationally symmetric or nonpolyhedral cells, a stochastic volume and surface area method that uses ray tracing is available. See page 2–182.

### ***A. Rotationally Symmetric Volumes and Areas***

The procedure for computing volumes and surface areas of rotationally symmetric bodies follows:

1. Determine the common axis of symmetry of the cell.<sup>114</sup> If there is none and if the cell is not a polyhedron, MCNP cannot compute the volume (except stochastically) and the area of each bounding surface cannot be computed on the side of the asymmetric cell.
2. Convert the bounding surfaces to q-form:

$$ar^2 + br + cs^2 + ds + e = 0 \quad ,$$

where  $s$  is the axis of rotational symmetry in the  $r$ - $s$  coordinate system. All MCNP surfaces except tori are quadratic surfaces and therefore can be put into q-form.

3. Determine all intersections of the bounding surfaces with each other in the  $r$ - $s$  coordinate system. This procedure generally requires the solution of a quartic equation.<sup>22</sup> For spheres, ellipses, and tori, extra intersection points are added so that

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these surfaces are not infinite. The list of intersections are put in order of increasing s-coordinate. If no intersection is found, the surface is infinite; its volume and area on one side cannot be computed.

4. Integrate over each bounding surface segment between intersections:

$$V = \pi \int r^2 ds \quad \text{for volumes;}$$

$$A = 2\pi \int r \sqrt{1 + \left(\frac{dr}{ds}\right)^2} ds \quad \text{for surface areas.}$$

A bounding surface segment lies between two intersections that bound the cell of interest.

A numerical integration is required for the area of a torroidal surface; all other integrals are directly solved by integration formulas. The sense of a bounding surface to a cell determines the sign of  $V$ . The area of each surface is determined cell-by-cell twice, once for each side of the surface. An area will be calculated unless bounded on both sides by asymmetric or infinite cells.

***B. Polyhedron Volumes and Areas***

A polyhedron is a body bounded only by planes that can have an arbitrary orientation. The procedure for calculating the volumes and surface areas of polyhedra is as follows:

1. For each facet side (planar surface), determine the intersections  $(r_i, s_i)$  of the other bounding planes in the  $r$ - $s$  coordinate system. The  $r$ - $s$  coordinate system is redefined for each facet to be an arbitrary coordinate system in the plane of the facet.
2. Determine the area of the facet:

$$a = \frac{1}{2} \sum (s_{i+1} - s_i)(r_{i+1} + r_i) \quad ,$$

and the coordinates of its centroid,  $r_c, s_c$ :

$$r_c = 1/(6a) \sum (s_{i+1} - s_i)(r_{i+1}^2 + r_{i+1}r_i + r_i^2) \quad .$$

$$s_c = 1/(6a) \sum (r_{i+1} - r_i)(s_{i+1}^2 + s_{i+1}s_i + s_i^2) \quad .$$

The sums are over all bounding edges of the facet where  $i$  and  $i + 1$  are the ends of the bounding edge such that, in going from  $i$  to  $i + 1$ , the facet is on the right side. As with rotationally symmetric cells, the area of a surface is determined cell-by-cell twice, once for each side. The area of a surface on one side is the sum over all facets on that side.

3. The volume of a polyhedron is computed by using an arbitrary reference plane. Prisms are projected from each facet normal to the reference plane, and the volume of each prism is  $V = da \cos \theta$  where  
 $d$  = distance from reference plane to facet centroid;  
 $a$  = facet area; and  
 $\theta$  = angle between the external normal of the facet and the positive normal of the reference plane.

The sum of the prism volumes is the polyhedron cell volume.

### C. *Stochastic Volume and Area Calculation*

MCNP cannot calculate the volumes and areas of asymmetric, nonpolyhedral, or infinite cells. Also, in very rare cases, the volume and area calculation can fail because of roundoff errors. For these cases a stochastic estimation is possible by ray tracing. The procedure is as follows:

1. Void out all materials in the problem (VOID card).
2. Set all nonzero importances to one and all positive weight windows to zero.
3. Use a planar source with a source weight equal to the surface area to flood the geometry with particles. This will cause the particle flux throughout the geometry to statistically approach unity. Perhaps the best way to do a stochastic volume estimation is to use an inward-directed, biased cosine source on a spherical surface with weight equal to  $\pi r^2$ .
4. Use the cell flux tally (F4) to tabulate volumes and the surface flux tally (F2) to tabulate areas. The cell flux tally is inversely proportional to cell volume. Thus in cells whose volumes are known, the unit flux will result in a tally of unity and in cells whose volume is uncalculated, the unit flux will result in a tally of volumes. Similarly, the surface flux tally is inversely proportional to area so that the unit flux will result in a tally of unity wherever the area is known and a tally of area wherever it is unknown.

## X. *PLOTTER*

The MCNP plotter draws cross-sectional views of the problem geometry according to commands entered by the user. See Appendix B for the command vocabulary and examples of use. The pictures can be drawn on the screen of a terminal or on some local or remote hard copy graphics device, as directed by the user. The pictures are drawn in a square viewport on the graphics device. The mapping between the viewport and the portion of the problem space to be plotted, called the window, is user-defined. A plane in problem space, the plot plane, is defined by specifying an origin  $\vec{r}_o$  and two perpendicular basis vectors  $\vec{a}$  and  $\vec{b}$ . The size of the window in the plot plane is defined by specifying two extents. The picture appears in the viewport with