Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and realistic spectral function for quasielastic region

M.V. Ivanov\textsuperscript{1,2} A.N. Antonov\textsuperscript{1} M.B. Barbaro\textsuperscript{3} J.A. Caballero\textsuperscript{4} G.D. Megias\textsuperscript{4} R. González-Jiménez\textsuperscript{4} E. Moya de Guerra\textsuperscript{2} J.M. Udías\textsuperscript{2}

\textsuperscript{1}Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria

\textsuperscript{2}Grupo de Física Nuclear, Departamento de Física Atómica, Molecular y Nuclear, Universidad Complutense de Madrid, Madrid, Spain

\textsuperscript{3}Dipartimento di Fisica, Università di Torino and INFN, Sezione di Torino, Torino, Italy

\textsuperscript{4}Departamento de Física Atómica, Molecular y Nuclear, Universidad de Sevilla, Sevilla, Spain
1. Theoretical scheme

2. Test versus electron scattering

3. Neutrino scattering
1. Theoretical scheme

2. Test versus electron scattering

3. Neutrino scattering
In most neutrino experiments, the interactions of the neutrinos occur with nucleons bound in nuclei. The influence of nucleon interactions on the response of nuclei to neutrino probes must then be considered, ideally in a model independent way. Model predictions for these reactions involve many different effects (nuclear correlations, interactions in the final state, possible modification of the nucleon and nucleon resonances properties inside the nuclear medium) that presently cannot be computed in a unambiguous and precise way. This is particularly true for the channels where neutrino interactions take place by means of excitation of a nucleon resonance and ulterior production of mesons.
One way of avoiding model-dependencies is to use the nuclear response to other leptonic probes, such as electrons, under similar conditions than the neutrino experiments. In this work we have compared the predictions for neutrino-induced CC $\pi^+$ production cross sections on mineral oil (CH$_2$) obtained using the superscaling analyses with the MiniBooNE data

\[ A.A. \text{ Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys. Rev. D 83, 052007 (2011).} \]

and for $\nu_\mu$-$^{12}$C inclusive charged current double differential cross sections with T2K data

\[ K. \text{ Abe et al. (T2K Collaboration), Phys. Rev. D 87, 092003 (2013).} \]

This is justified on the success of the scaling ideas and the observation of superscaling in electron-nucleus scattering.
The method relies on the superscaling properties of the electron scattering data: at sufficiently high momentum transfer the inclusive differential \((e, e')\) cross sections, divided by a suitable function which takes into account the single nucleon content of the problem, depends only upon one kinematical variable, the scaling variable (this behavior is called scaling of first kind) and the resulting function is roughly the same for all nuclei (scaling of second kind). When both kinds of scaling are fulfilled the cross section is said to superscale.
More specifically the scaling function

\[ f = \frac{d^2 \sigma / d \Omega dk'}{S(q, \omega)}, \]  

(1)

being \( S \) related to the single nucleon cross section, becomes, for large \( q \), a function only of the scaling variable

\[ \psi_{QE} = \pm \sqrt{\frac{1}{2T_F}} \left( q \sqrt{1 + \frac{1}{\tau}} - \omega - 1 \right), \]  

(2)

where \( T_F \) is the Fermi kinetic energy, \( 4m_N^2 \tau = q^2 - \omega^2 \) and the \(-\) (\(+\)) sign corresponds to energy transfers lower (higher) than the QEP (\( \psi = 0 \)).
Indeed the analyses of the world data on inclusive electron-nucleus scattering confirmed the observation of this phenomenon and thus allowed to extract a universal nuclear response to weak interacting probes.
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and . . .
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and 

\[ f_{\text{RFG}}(\psi) = \frac{3}{4} (1 - \psi^2) \theta(1 - \psi^2) \]
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and...
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and...
The superscaling analysis has been extended to the first resonance peak in


where the contribution of the $\Delta$ has been (approximately) isolated in the experimental data by subtracting the quasielastic scaling contribution from the total experimental cross sections; the reduced cross section has been studied as a function of a new scaling variable

$$
\psi_{\Delta} = \pm \sqrt{\frac{1}{2T_F} \left( q\sqrt{\rho + \frac{1}{\tau}} - \omega \rho - 1 \right)},
$$

(3)

which accounts for the inelasticity through the quantity

$$
\rho = 1 + (m^2_{\Delta} - m^2_N)/(4\tau m^2_N).
$$
The results show that also in this region superscaling is working quite well and therefore a second superscaling function, $f^\Delta(\psi_\Delta)$, can be extracted from the data to account for the nuclear dynamics. Clearly this approach can work only at $\psi_\Delta < 0$, since at $\psi_\Delta > 0$ other resonances and the tail of the deep-inelastic scattering start contributing.
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and...

\[ f_{\text{RFG}}(\psi_\Delta) = \frac{3}{4}(1 - \psi_\Delta^2)\theta(1 - \psi_\Delta^2) \]
1. Theoretical scheme

2. Test versus electron scattering

3. Neutrino scattering
\( ^{12}\text{C}, \varepsilon_e = 680 \text{ MeV}, \theta_e = 36.000^\circ, q_{QE} \approx 402.5 \text{ MeV} \)

\( ^{12}\text{C}, \varepsilon_e = 1930 \text{ MeV}, \theta_e = 16.000^\circ, q_{QE} \approx 536.3 \text{ MeV} \)

\( ^{12}\text{C}, \varepsilon_e = 2130 \text{ MeV}, \theta_e = 16.000^\circ, q_{QE} \approx 593.9 \text{ MeV} \)

\( ^{12}\text{C}, \varepsilon_e = 1108 \text{ MeV}, \theta_e = 37.500^\circ, q_{QE} \approx 674.6 \text{ MeV} \)
1. Theoretical scheme

2. Test versus electron scattering

3. Neutrino scattering
To obtain the results of applying the SuSA and RFG $\Delta$-scaling function to the calculations of neutrino-induced charged-current charged pion production, we follow the formalism given in


The charge-changing neutrino cross section in the target laboratory frame is given in the form

$$
\left[ \frac{d^2\sigma}{d\Omega dk'} \right]_\chi \equiv \sigma_0 F^2_\chi; \quad \sigma_0 \equiv \frac{(G \cos \theta_c)^2}{2\pi^2} \left[ k' \cos \tilde{\theta}/2 \right]^2 \tag{4}
$$

where $\chi = +$ for neutrino-induced reactions (for example, $\nu_l + n \rightarrow \ell^- + p$, where $\ell = e, \mu, \tau$) and $\chi = -$ for antineutrino-induced reactions (for example, $\bar{\nu}_l + p \rightarrow \ell^+ + n$), $G = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant and $\theta_c$ is the Cabibbo angle ($\cos \theta_c = 0.9741$),

$$
\tan^2 \tilde{\theta}/2 \equiv \frac{|Q^2|}{\nu_0}, \quad \nu_0 \equiv (\epsilon + \epsilon')^2 - q^2 = 4\epsilon \epsilon' - |Q^2|.
$$
The function $\mathcal{F}_\chi^2$ depends on the nuclear structure and can be written as:

$$\mathcal{F}_\chi^2 = [\hat{V}_{CC}R_{CC} + 2\hat{V}_{CL}R_{CL} + \hat{V}_{LL}R_{LL} + \hat{V}_TR_T] + \chi[2\hat{V}_{T'}R_{T'}]$$

that is, as a generalized Rosenbluth decomposition having charge-charge (CC), charge-longitudinal (CL), longitudinal-longitudinal (LL) and two types of transverse ($T, T'$) responses ($R$'s) with the corresponding leptonic kinematical factors ($V$'s). The nuclear response functions in $\Delta$-region are expressed in terms of the nuclear tensor $W^{\mu\nu}$ in the corresponding region times the single-nucleon response.
The nuclear response functions $R_i$ can be expressed in terms of the nuclear tensor $W^{\mu \nu}$. For the excitation of a stable $\Delta$ resonance the RFG hadronic tensor turns out to be:

$$[W^{\mu \nu}]^{\Delta} = \frac{1}{2} \frac{3N}{4m_N \eta_F^3 \kappa} f_{RFG}(\psi_\Delta) U^{\mu \nu}_\Delta, \quad (5)$$

where $\kappa = q/2m_N$ and $\eta_F = \sqrt{\xi_F(\xi_F + 2)} = k_F/m_N$ are the dimensionless transferred and Fermi momentum, respectively, and the “single-nucleon” tensor $U^{\mu \nu}_\Delta$ embeds the information about the $N - \Delta$ transition$^1$.

The basic expressions used to calculate the “single-nucleon” tensor $U^{\mu \nu}_\Delta$ are given in


$^1$Actually in a fully relativistic framework the single nucleon physics cannot be exactly disentangled from the many-body part of the problem and the tensor $U^{\mu \nu}_\Delta$ contains corrections due to the medium.
A convenient parametrization of the single-nucleon $W^+ n \rightarrow \Delta^+$ vertex is given in terms of eight form-factors: four vector ($C_{3,4,5,6}^V$) and four axial ($C_{3,4,5,6}^A$) ones. Vector form factors have been determined from the analysis of photo and electro-production data, mostly on a deuteron target, considered as a good approximation to free nucleons to these purposes. Among the axial form factors the most important contribution comes from $C_5^A$. The form factor $C_6^A$, whose contribution to the differential cross section vanishes for massless leptons, can be related to $C_5^A$ due to the partial conservation of the axial current. Since there are no other theoretical constraints for $C_{3,4,5}^A(q^2)$, they have to be fitted to data.
In our calculations we use for vector and axial form-factors two different parameterizations: the one given in


where deuteron effects were evaluated (authors estimated that the latter reduce the cross section by 10%), we call it “PR1”, and the one from


called by us “PR2”. 
With these ingredients, we obtain the cross section for CC $\Delta^{++}$ and $\Delta^{+}$ production on proton and neutron, respectively. Once produced, the $\Delta$ decays into $\pi N$ pairs. For the amplitudes $A$ of pion production the following isospin decomposition applies

\[
A(\nu_l p \rightarrow l^- p \pi^+) = A_3, \\
A(\nu_l n \rightarrow l^- n \pi^+) = \frac{1}{3} A_3 + \frac{2\sqrt{2}}{3} A_1, \\
A(\nu_l n \rightarrow l^- p \pi^0) = -\frac{\sqrt{2}}{3} A_3 + \frac{2}{3} A_1,
\]

with $A_3$ being the amplitude for the isospin 3/2 state of the $\pi N$ system, predominantly $\Delta$, and $A_1$, the amplitude for the isospin 1/2 state which is not considered here.
\[ \pi^+ \text{ production from } \Delta \text{ resonance region of neutrino-induced charged-current } \nu_\mu - \text{CH}_2 \]

(MiniBooNE experiment)
Firstly, I will show the results for the double-differential cross section results of the $\pi^+$ production from $\Delta$ resonance region of neutrino-induced charged-current $\nu_\mu$–CH$_2$ reaction averaged over the neutrino flux $\Phi(\epsilon_\nu)$, namely

$$\frac{d^2\sigma}{dT_\mu \, d\cos \theta} = \frac{1}{\Phi_{\text{tot}}} \int \left[ \frac{d^2\sigma}{dT_\mu \, d\cos \theta} \right]_{\epsilon_\nu} \Phi(\epsilon_\nu) \, d\epsilon_\nu,$$

where $T_\mu$ and $\theta$ are correspondingly the kinetic energy and scattering angle of the outgoing muon, $\epsilon_\nu$ is the neutrino energy and $\Phi_{\text{tot}}$ is the total integrated $\nu_\mu$ flux factor for the MiniBooNE experiment. In each figure the results have been averaged over the corresponding angular bin of $\cos \theta$. 
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and...
$\frac{d^2\sigma}{dT_\mu}(\cos q) [10^{-42} \text{ cm}^2/\text{MeV}]$

- $0.15 < \cos q < 0.20$
- $0.00 < \cos q < 0.05$
- $-0.20 < \cos q < -0.10$
- $-0.50 < \cos q < -0.40$

Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and...
The SuSA approach provides neutrino-nucleus cross-section predictions, based on the observed nuclear response to electron projectiles under similar kinematics as the neutrinos considered in this work. In this way, the SuSA scaling function and corresponding cross-section predictions, incorporate effect of final state interaction, the properties of the $\Delta$ resonance in the nuclear medium, etc. SUSA predictions are in good agreement with the MiniBooNE experimental data for pionic cross-section.
$\nu_\mu$ inclusive charged current double differential cross sections

(T2K experiment)
Charged current inclusive neutrino cross sections: Superscaling extension to the pion production and ...
So, we can conclude that the idea of the SuSA approach for the $\Delta$-region (extracted from electron scattering experiments) and natural orbitals (NO+FSI) scaling function including final state interaction for the QE-region and their extension to neutrino processes can be very useful in predicting of $\nu_\mu$ inclusive charged current cross sections.
THANK YOU VERY MUCH FOR YOUR ATTENTION !!!