Study on breakup mechanism of unstable nuclei with CDCC

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Introduction

- Nuclear Reaction
  - Breakup process
  - Stripping process
  - Transfer process

- Breakup process
- Stripping process
- Transfer process

- Shell evolution & deformation
- Island of inversion

- Valence neutrons in s or p
- Two-neutron halo
- Neutron Number
- Proton Number

- N=2
- Z=2
- N=8
- Z=8
- N=20

- 4He, 6He, 8He, 11Li, 12C, 14Be, 16O, 20Ne, 24Mg, 28Si, 30Mg, 32Mg, 34Mg, 27F, 29F, 31F
Breakup

- Breakup reactions are useful for investigating halo structure.

Observable
- Breakup cross section for $E_x$
  - Resonance states
  - Excited modes
- Momentum distribution
  - Particle spin (core)
  - Orbit of valence neutron

- To extract properties of halo, an accurate method for describing breakup processes is highly desired.

- **CDCC** is one of a reliable method for treating breakup processes.
Continuum Discretized Coupled Channels (CDCC)

- Breakup is effects of coupling to continuum states.
- In CDCC, breakup continuum states are assumed as a finite number of discretized states.

**Average method:**

\[ \phi_i = N_i \int_{E_i}^{E_{i-1}} \psi(E) dE \quad \psi(E) : \text{continuum w.f.} \]

**Pseudo-states method:**

\[ \phi_i = \sum_{j=1}^{N} C_{j}^{(i)} \varphi_j \quad \varphi_j : \text{Basis function} \]
One neutron removal of $^{19}$C


$^{19}$C + p → $^{18}$C + n + p

81 MeV/nucl

$^{18}$C*(3rd ex.)

- The aim of this experiment is to determine $I^\pi$ of the 3rd excited state of $^{18}$C.
- The transverse-momentum distribution of the 3rd excited state of $^{18}$C was measured using coincidences with the $\gamma$ rays.
Momentum Distribution of $^{18}$C

Ground state w.f. of $^{19}$C

$$\phi(^{19}C) = \alpha \left[ \phi_3(I^{\pi}) \otimes \chi_j \right]_{1/2^+}$$

$$j = l + 1/2$$

Solid line:

$I^{\pi} = 0^+$ or $1^+$ for $l = 0$

Dashed line:

$I^{\pi} = 2^+$ or $3^+$ for $l = 2$

Nuclear Data Evaluation
This work has been performed at Hokkaido Univ (JCPRG) and Kyushu Univ.

- **Background**
  - Lithium is an important element for a tritium breeding material in DT fusion reactors.
  - Accurate nuclear data of nucleon induced reactions on $^6,^7\text{Li}$ are required.

- **CDCC calculation**
  - $n + ^6\text{Li}$ scattering
  - $^6\text{Li}$ is described as $d + ^4\text{He}$ model.
  - Coupling potentials between $n$ and $^6\text{Li}$ are calculated by folding model with JLM interaction.

$n + ^6\text{Li} \text{ Breakup}$

- Neutron emitted spectrum corresponds to the excited structure of $^6\text{Li}$.
- CDCC is also useful for nuclear data evaluation.
Breakup Reaction of $^6$He

- $^6$He
- A typical example of two-neutron halo
- Described by $n + n + ^4$He three-body model


Nuclear Breakup

$^6$He + $^{12}$C at 18MeV
($N_R = 1.0$, $N_I = 0.5$)

Nuclear & Coulomb Breakup

$^6$He + $^{209}$Bi at 22.5 MeV

$0^+$ and $2^+$ coupling

$0^+$, $1^-$ and $2^+$ coupling
$^6\text{He}+^{12}\text{C}$, $^{208}\text{Pb} @ 240 \text{ MeV/nucl.}$

Exp. data from PRC59, 1252 (1999), T. Aumann et al.

$^6\text{He}+^{12}\text{C} @ 240 \text{ MeV/nucl.}$

Nuclear BU

$^6\text{He}+^{208}\text{Pb} @ 240 \text{ MeV/nucl.}$

Coulomb BU

Exp. data from PRC59, 1252 (1999), T. Aumann et al.
Eikonal Reaction Theory


**Eikonal reaction theory is an approach to calculate stripping cross section within CDCC framework.**

\[
\sigma_{2n\text{str}} = \int \, db \langle \phi_0 | |S_c|^2 (1 - |S_n|^2) (1 - |S_n|^2) | \phi_0 \rangle
\]

**\( {^6}\text{He} + {^{12}}\text{C} \) scattering @ 240MeV/nucl.**

**\( {^6}\text{He} + {^{209}}\text{Pb} \) scattering @ 240MeV/nucl.**

Exp. data from PRC59, 1252 (1999), T. Aumann et al.
$^6\text{Li} \ (n+p+^4\text{He}), \ ^{16}\text{C} \ (n+n+^{14}\text{C})$

Reaction cross section of $^{14,15,16}\text{C}$

Watanabe, TM, Minomo, Ogata, Yahiroyo, PRC86, 031601(R) (2012)

TM, Yahiroyo, PRC90, 041602(R) (2014)
Breakup reactions are useful for investigating unstable nuclei.

CDCC and ERT are one of reliable method for treating breakup process of two- and three-body projectiles accurately.

CDCC is applicable to not only study on unstable nuclei but also evaluation of nuclear data.
Collaborators

- **Hokkaido Univ.**: K. Kato, Y. Hirabayashi, D. Ichinkhorloo, M. Aikawa
- **Chiba Keizai Univ.**: Y. Iseri
- **Tokyo Inst. of Tech.**: S. Chiba, T. Nakamura, Y. Kondo
- **RCNP**: K. Ogata, K. Minomo
- **Kyushu Univ.**: M. Yahirot, S. Watanabe, Y. Watanabe, H. Guo
BACKUP
Three-body Breakup Reaction
The projectile breaks up into 2 particles.
Projectile (2-body) + target (1-body) → 3-body breakup reaction
Ex.) d, $^6$Li, $^{11}$Be, $^8$B, $^{15}$C, etc..

One-neutron halo

Four-body Breakup Reaction
The projectile breaks up into 3 particles.
Projectile (3-body) + target (1-body) → 4-body breakup reaction
Ex.) $^6$He, $^{11}$Li, $^{14}$Be, etc..

Two-neutron halo
Eikonal Reaction Theory

\[ S = S_n S_c \]

\[ \sigma_n = \int d\Omega \langle \phi_0 | |S_c|^2 (1 - |S_n|^2) |\phi_0 \rangle \]

Eikonal Reaction Theory (ERT)

\[ S_c (S_n) \text{ is estimated by solving CDCC equation without } U_c (U_n). \]

\[ \begin{bmatrix} -\frac{\hbar^2}{2\mu_R} \nabla^2_R + h_r + U_n(r, R) - E \end{bmatrix} \Psi(r, R) = 0 \quad \text{for } S_n \]

\[ \begin{bmatrix} -\frac{\hbar^2}{2\mu_R} \nabla^2_R + h_r + U_c(r, R) - E \end{bmatrix} \Psi(r, R) = 0 \quad \text{for } S_c \]
Discretization Method (1)

**Average method** (momentum-bin, energy-bin)

Two-body system

\[ \psi(k, r) \]

Threshold

Bound state

Three-body system

Application of the average method is not easy for scattering of three-body projectiles, because it requires the exact three-body continuum states.

Discretization Method (2)

**Pseudo-state method**

Wave function of discretized state

\[ \hat{\phi}_i(r) = \sum_{n=1}^{N} C_n^{(i)} \varphi_n(r) \]

\( \varphi_n(r) : \text{Basis function (Gaussian, HH, ...) } \)

\( C_n^{(i)} \) can be obtained by diagonalizing \( H \) of the projectile

\[
\begin{pmatrix}
    H_{nn'} & \epsilon_i \\
    N_{nn'} &
\end{pmatrix}
\begin{pmatrix}
    C_n^{(i)}
\end{pmatrix} = 0
\]

*The pseudo-state method can be easily applicable to four-body breakup reactions.*
One neutron removal of $^{19}\text{C}$


$^{19}\text{C} + p \rightarrow ^{18}\text{C} + n + p$

$^{19}\text{C}$

$^{18}\text{C}$

The transverse momentum distribution of $3^{rd}$ excited state of $^{18}\text{C}$ core was measured.

$\phi(^{19}\text{C}) = \alpha [\phi_3 \otimes \chi_{\ell}]_{1/2^+}$

Solid: $\phi_3 : 0^+ / 1^+ \text{ for } \ell = 0$

Dashed line: $\phi_3 : 2^+ / 3^+ \text{ for } \ell = 2$

81 MeV/nucl

$1^{1}\text{H}(^{19}\text{C}, ^{18}\text{C} + \gamma)$

$E_\gamma = 2.4 \text{ MeV}$

$\gamma$

$P_x (\text{MeV}/c)$

$\frac{d\sigma}{dp_x} (\text{a.u.})$
Transverse Momentum of $^{17}\text{C}$

$E_\gamma = 0.21 \text{ MeV}$

In this analysis, we assume that the core remains after one neutron is removed.

$J^\pi = 1/2^+$ for 0.21 MeV state

$J^\pi = 5/2^+$ for 0.33 MeV state
Breakup Cross Section

Breakup cross sections calculated by CDCC are discrete in the internal energy of the projectile.

How to calculate the continuous breakup cross section

Results of CDCC

$^{6}\text{He} + ^{12}\text{C}$ at 240 MeV/nucl.

PRC59, 1252(1999), T. Aumann et al.
Complex-Scaling Method (CSM)

Complex-scaling operator: \( U^\theta \)

\[
U^\theta f(r) = e^{i\frac{3}{2}\theta} f(re^{i\theta})
\]

Coordinate: \( r \rightarrow re^{i\theta} \)

Momentum: \( k \rightarrow ke^{-i\theta} \)

Useful for searching many-body resonances

Green’s function with Complex-Scaling Method (CDCS Green’s function)

\[
G^{(-)}(E, \xi, \xi') = \frac{1}{E - H - i\epsilon} \approx \sum_{\nu} U^{-\theta} \frac{|\Phi_{\nu}^\theta\rangle \langle \tilde{\Phi}_{\nu}^\theta|}{E - E_{\nu}^\theta} U^\theta
\]
**Smoothing with CSM**

**Discrete T matrix**

\[ T_i = \langle \Phi_i \chi_C^{(-)}(R) | V | \Psi^{(+)}(\xi, R) \rangle \]

**Continuous T matrix**

\[ T(E) = \langle \psi^{(-)}(E, \xi) \chi_C^{(-)}(R) | V | \Psi^{(+)}(\xi, R) \rangle \]

\[ \frac{d\sigma}{dE} = \int T^\dagger(E') T(E')\delta(E - E')dE' = \frac{1}{\pi} \text{Im}\mathcal{R}(E) \]

\[ \mathcal{R}(E) = \int d\xi d\xi' \langle \Psi^{(+)}(\xi, R) | V^* | \chi_C^{(-)}(R) \rangle \mathcal{G}^{(-)}(E, \xi, \xi') \langle \chi_C^{(-)}(R) | V | \Psi^{(+)}(\xi, R) \rangle \]

**Green's function with Complex-Scaling Method (CDCS Green's function)**

\[ \mathcal{G}^{(-)}(E, \xi, \xi') \approx \sum_{\nu} \sum_{i,j} |\Phi_i\rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_\nu \rangle \langle \Phi_\nu | U^\theta | \Phi_j \rangle}{E - E_\nu^\theta} \langle \Phi_j| \]

\[ \mathcal{R}(E) = \sum_{\nu} \sum_{i,j} \langle \Psi^{(+)}| V^* | \chi_C^{(-)} \Phi_i \rangle \frac{\langle \Phi_i | U^{-\theta} | \Phi_\nu \rangle \langle \Phi_\nu | U^\theta | \Phi_j \rangle}{E - E_\nu^\theta} \langle \Phi_j \chi_C^{(-)} | V | \Psi^{(+)} \rangle \]

*T-matrix calculated by CDCC*
New description of differential breakup cross section

\[
\frac{d\sigma}{dE} = \frac{1}{\pi} \text{Im} \sum_{\nu} \sum_{i,j} T_{i}^{\text{CDCC}} T_{j}^{\text{CDCC}} \left[ \frac{\langle \Phi_{i} | U^{-\theta} | \Phi_{\nu}^{\theta} \rangle \langle \tilde{\Phi}_{\nu}^{\theta} | U^{\theta} | \Phi_{j} \rangle}{E - E_{\nu}^{\theta}} \right]
\]

\[E_{\text{rel}} [\text{MeV}] \]

\[\sigma [\text{mb}]\]

\[\frac{d\sigma}{dE_{\text{rel}}} [\text{mb/MeV}]\]

$^6\text{He} + ^{12}\text{C}$ scattering at 229.8 MeV
Convergence

1. Convergence of \( T \)-matrix elements calculated by CDCC

\[
1 \approx \sum_i |\phi_i\rangle \langle \phi_i| \quad \rightarrow \quad \Psi(\xi, \mathbf{R}) \approx \sum_i \phi_i(\xi) \chi_i(\mathbf{R})
\]

2. Convergence of Green’s function with CSM.

\[
1 \approx \sum_\nu |\phi^\theta_\nu\rangle \langle \phi^\theta_\nu| \quad \rightarrow \quad \mathcal{G}(E, \xi, \xi') \approx \sum_\nu U^{-\theta} \frac{|\phi^\theta_\nu\rangle \langle \phi^\theta_\nu|}{E - E^\theta_\nu} U^\theta
\]

We should confirm the convergence with extending the model space

<table>
<thead>
<tr>
<th></th>
<th>The number of bases</th>
<th>Gaussian range max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set I</td>
<td>10</td>
<td>10 fm</td>
</tr>
<tr>
<td>Set II</td>
<td>15</td>
<td>20 fm</td>
</tr>
<tr>
<td>Set III</td>
<td>20</td>
<td>50 fm</td>
</tr>
</tbody>
</table>
Convergence of $T$-matrix ($2^+$)

The $T$-matrix calculated with set I gives good convergence.

The T-matrix calculated with set I gives good convergence
The result with set II gives good convergence for Green’s function
Application to Nuclear Engineering

- **Background**
  - Lithium is an important element relevant to not only a tritium breeding material in DT fusion reactors but also a candidate for target material in the intense neutron source of IFMIF.
  - Accurate nuclear data of nucleon induced reactions on $^{6,7}$Li are currently required for incident energies up to 150 MeV.
  - Statistical model is not applicable to Li scattering because the *clustering structure* is important for Li.

- **CDCC calculation**
  - $^{6,7}$Li structure
    - $^6$Li = $^4$He + d
    - $^7$Li = $^4$He + t
  - Nucleon-$^{6,7}$Li interaction
    - Single folding model with JLM

This work has been performed at Hokkaido Univ (JCPRG) and Kyushu Univ.
$^{12}$C ($^4$He+$^4$He+$^4$He) scattering

- First application of CDCC to four-body system
  - Breakup effects on elastic and inelastic
  - Coupling to cluster states (resonance)

Y. Sakuragi et al., PTP Suppl.89, 136 (1986)
Phenomenological 3-body force

\[ V_{nnn}^{(v)}(y_1, r_1 \to \infty) = 0 \]

Volume Type Gaussian

\[ V_{nnn}^{(v)}(y_1, r_1) = V_0 e^{-y_1^2/y_0^2} e^{-r_1^2/r_0^2} \]

Surface Type Gaussian

\[ V_{nnn}^{(s)}(y_1, r_1) = V_0 y_1^2 e^{-y_1^2/y_0^2} e^{-r_1^2/r_0^2} \]

\( \rightarrow \) strong d-wave mixing in ground state of \(^{16}\text{C}\)
Elastic scattering

Calculated by Guo (Kyushu Univ.)
Nucleon emission (BU of Li)

Neutron emitted spectrum represents the excited structure of $^6\text{Li}$, and CDCC calculation well reproduce the data.
Odd-even staggering of $\sigma_R$

$\sigma(A) < \sigma(A+1) > \sigma(A+2)$

2-body system: one-neutron halo system
3-body system: (one-neutron halo) + $n$
$^{14}\text{C} + n, \quad ^{14}\text{C} + n + n + n$ model

Two-body model Hamiltonian of $^{15}\text{C}$

$$H_{^{15}\text{C}} = -\frac{\hbar^2}{2\mu_y} \nabla_y^2 + V_{nc}(y)$$

Three-body model Hamiltonian of $^{16}\text{C}$

$$H_{^{16}\text{C}} = -\frac{\hbar^2}{2\mu_r} \nabla_r^2 - \frac{\hbar^2}{2\mu_y} \nabla_y^2 + V_{nc}(y_1) + V_{nc}(y_2) + V_{nn}(y_3) + V_{nnn}(r, y)$$

$V_{nc}$: central + LS + OCM \[1\]

$1/2^+$: $-1.215$ MeV

$5/2^+$: $-0.478$ MeV

$V_{nn}$: BonnA

$V_{nnn}$: Phenomenological 3-body force

[1] Hagino, and Sagawa, PRC75, 021301
$^{14}\text{C} + n + n$

- Three-body calculation w/o $V_{nn}$ and $V_{nnn}$
  - $H_{16\text{C}} = 2 \times H_{15\text{C}}$
  - $2n$ separation energy : 2.40 MeV (1.218 MeV x 2)
  - Configuration : $|\chi_{nc}(1s_{1/2})\rangle \otimes |\chi_{nc}(1s_{1/2})\rangle$  99%

- Three-body calculation w/o $V_{nnn}$
  - $2n$ separation energy : 3.219 MeV (EXP: 5.469 MeV)
  - Configuration : $|\chi_{nc}(1s_{1/2})\rangle \otimes |\chi_{nc}(1s_{1/2})\rangle$  79%
  - $|\chi_{nc}(0d_{5/2})\rangle \otimes |\chi_{nc}(0d_{5/2})\rangle$  16%

We introduce the phenomenological 3-body force to reproduce the binding energy.
Phenomenological 3-body force

Volume Type Gaussian

\[ V_{nnc}^{(v)}(y_1, r_1) = V_0 e^{-y_1^2/y_0^2} e^{-r_1^2/r_0^2} \]

Surface Type Gaussian

\[ V_{nnc}^{(s)}(y_1, r_1) = V_0 y_1^2 e^{-y_1^2/y_0^2} e^{-r_1^2/r_0^2} \]

\[ V_{nnc}(y_1, r_1 \to \infty) = 0 \]
Effect of 3-body force

\[ H_{15C}(r_n) = T_y + V_{nc}(y) + V_{nnc}(y, r_n) \]

Strong d-wave mixing in ground state of $^{16}\text{C}$ with surface-type $1s_{1/2}$.
Density of $^{16}$C

**Density distribution of $^{16}$C**

$^{14}$C-core: HFB calculation

- $<r_{rms}>^{1/2}(^{14}$C$) = 2.51$ fm
- $<r_{rms}>^{1/2}(^{15}$C$) = 2.87$ fm

**Valence neutron density of $^{16}$C**

Vol.: $<r_{rms}>^{1/2} = 2.80$ fm
- $(1s_{1/2})^2$: 72%, $(0d_{5/2})^2$: 14%

Sur.: $<r_{rms}>^{1/2} = 2.72$ fm
- $(1s_{1/2})^2$: 15%, $(0d_{5/2})^2$: 74%
Main configuration of valence two neutrons of $^{16}\text{C}$ is $(0d_{5/2})^2$. One neutron removal from $^{16}\text{C}$: $(1s_{1/2})^2 \sim 30\%$ [Yamaguchi et al., NPA724, 3(2006)]
One- and two-neutron removal

<table>
<thead>
<tr>
<th>Reaction process</th>
<th>Breakup</th>
<th>1n stripping</th>
<th>2n stripping</th>
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<tbody>
<tr>
<td>$^{16}\text{C}$</td>
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<td>$^{15}\text{C}$</td>
</tr>
</tbody>
</table>

ERT can calculate total breakup, 1n stripping, and 2n stripping cross sections.
$^6$He : $n + n + ^4$He

- $^6$He
  - A typical example of two-neutron halo
  - Described by $n + n + ^4$He three-body model

- Pseudo-state method
  - Gaussian Expansion Method
  - An accurate method of solving few-body problems.
  - Hamiltonian is diagonalized with Gaussian bases
  - Three Jacobi coordinates

Breakup Effects on Elastic

$^6\text{He}+^{12}\text{C}$ scattering at 18 MeV


$^6\text{He}+^{209}\text{Bi}$ scattering at 22.5 MeV


$^6\text{He}+^{209}\text{Bi}$ at 22.5 MeV

Nuclear & Coulomb Breakup

$^6\text{He}+^{12}\text{C}$ at 18 MeV
($N_R=1.0, N_I=0.5$)

Nuclear Breakup

$\sigma/\sigma_R$

$\theta_{c.m.}$ [deg]

no coupling

$0^+$ and $2^+$ coupling

$\sigma/\sigma_R$

$\theta_{c.m.}$ [deg]

no coupling

$0^+, 1^-$ and $2^+$ coupling