Deuteron–induced reactions, spin distributions and the surrogate method

Grégory Potel Aguilar (NSCL, LLNL)
Filomena Nunes (NSCL)
Ian Thompson (LLNL)

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We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:

- **Direct neutron transfer**: should be compatible with existing theories.
- **Elastic deuteron breakup**: “transfer” to continuum states.
- **Non elastic breakup** (direct transfer, inelastic excitation and compound nucleus formation): absorption above and below neutron emission threshold.
- **Important application in surrogate reactions**: obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation.
Historical background

breakup-fusion reactions

- Kerman and McVoy, Ann. Phys. 122 (1979) 197
- Controversy between Udagawa and Austern formalism left somehow unresolved.

Protons and $\alpha$ yields bombarding $^{209}$Bi with $^{12}$C and $^{16}$O

Britt and Quinton, Phys. Rev. 124 (1961) 877
let’s concentrate in the reaction \( A+d \rightarrow B(=A+n)+p \)

we are interested in the inclusive cross section, i.e., we will sum over all final states \( \phi_B^c \).
Neutron states in nuclei

- Neutron scattering states
- Weakly bound states
- Deeply bound states

Imaginary part of optical potential

Narrow single-particle scattering and resonances

Broad single-particle

Weakly bound states

Deeply bound states

Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1
Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final $\phi_B^c$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \left| \left\langle \chi_p \phi_B^c | V | \psi(+) \right\rangle \right|^2.$$

Sum over all channels, with the approximation $\psi(+) \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d} \rho(E_p) \times \sum_c \left\langle \chi_d \phi_d \phi_A | V | \chi_p \phi_B^c \right\rangle \delta(E - E_p - E_B^c) \left\langle \phi_B^c \chi_p | V | \phi_A \chi_d \phi_d \right\rangle$$

$\chi_d \rightarrow$ deuteron incoming wave, $\phi_d \rightarrow$ deuteron wavefunction,
$\chi_p \rightarrow$ proton outgoing wave $\phi_A \rightarrow$ target core ground state.
the imaginary part of the Green's function $G$ is an operator representation of the $\delta$–function,

$$
\pi \delta(E - E_p - E_B^c) = \lim_{\epsilon \to 0} \Im \sum_c \frac{\langle \phi_B^c | \phi_B^c \rangle}{E - E_p - H_B + i\epsilon} = \Im G
$$

$$
\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d} \rho(E_p) \Im \langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle
$$

- We got rid of the (infinite) sum over final states,
- but $G$ is an extremely complex object!
- We still need to deal with that.
If the interaction $V$ do not act on $\phi_A$

$$\langle \chi_d \phi_d \phi_A | V | \chi_p \rangle G \langle \chi_p | V | \phi_A \chi_d \phi_d \rangle$$

$$= \langle \chi_d \phi_d | V | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | V | \chi_d \phi_d \rangle$$

$$= \langle \chi_d \phi_d | V | \chi_p \rangle G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle,$$

where $G_{opt}$ is the optical reduction of $G$

$$G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus $G_{opt}$ are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations.
the imaginary part of $G_{opt}$ splits in two terms:

$$\Im G_{opt} = -\pi \sum_{k_n} |\chi_n\rangle \delta \left( E - E_p - \frac{k_n^2}{2m_n} \right) \langle \chi_n | + G_{opt}^\dagger W_{An} G_{opt},$$

we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$

cross sections for neutron capture and elastic breakup

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{\text{capture}} = -\frac{2}{\hbar \nu_d} \rho(E_p) \langle \psi_n | W_{An} | \psi_n \rangle,$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \bigg|_{\text{breakup}} = -\frac{2}{\hbar \nu_d} \rho(E_p) \rho(E_n) |\langle \chi_n \chi_p | V | \chi_d \phi_d \rangle|^2,$$
2–step process (post representation)

step 1: breakup

$\langle \chi_p | V | \phi_{AX} d\phi_d \rangle$

Step 1: non elastic breakup

step 2: propagation of n in the field of A

Step 2: to detector

- p: non elastic breakup
- p: elastic breakup

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The interaction $V$ can be taken either in the *prior* or the *post* representation,

- Austern (post) $\rightarrow V \equiv V_{\text{post}} \sim V_{\text{pn}}(r_{\text{pn}})$ (recently revived by Moro and Lei, University of Sevilla)
- Udagawa (prior) $\rightarrow V \equiv V_{\text{prior}} \sim V_{\text{An}}(r_{\text{An}}, \xi_{\text{An}})$

in the *prior* representation, $V$ can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$
\frac{d^2 \sigma}{d \Omega_p dE_p} \bigg|_{\text{post}}^{\text{post}} = - \frac{2}{\hbar \nu_d} \rho(E_p) \left[ \Im \left\langle \psi_{n_{\text{prior}}}^n | W_{\text{An}} | \psi_{n_{\text{prior}}}^n \right\rangle 
+ 2 \Re \left\langle \psi_{n_{\text{NON}}}^n | W_{\text{An}} | \psi_{n_{\text{prior}}}^n \right\rangle + \left\langle \psi_{n_{\text{NON}}}^n | W_{\text{An}} | \psi_{n_{\text{NON}}}^n \right\rangle \right],
$$

where $\psi_{n_{\text{NON}}}^n = \langle \chi_p | \chi_d \phi_d \rangle$.

The nature of the 2–step process depends on the representation
The neutron wavefunctions

\[ |\psi_n\rangle = G_{opt} \langle \chi_p| V |\chi_d\phi_d\rangle \]

can be computed for any neutron energy.

These wavefunctions are not eigenfunctions of the Hamiltonian

\[ H_{An} = T_n + \Re(U_{An}) \]
Breakup above neutron–emission threshold

proton angular differential cross section

[^93]Nb (d,p), $E_d=15$ MeV

$\theta$

$E_p=9$ MeV  $E_n=3.8$ MeV
Let's consider the limit \( W_{An} \to 0 \) (single–particle width \( \Gamma \to 0 \)). For an energy \( E \) such that \( |E - E_n| \ll D \), (isolated resonance)

\[
G_{opt} \approx \lim_{W_{An} \to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};
\]

with \( |\phi_n\rangle \) eigenstate of \( H_{An} = T_n + \Re(U_{An}) \)

\[
\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An} \to 0} \langle \chi_d \phi_d | V | \chi_p \rangle \\
\times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle \chi_p | V | \chi_d \phi_d \rangle,
\]

we get the direct transfer cross section:

\[
\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\langle \chi_p \phi_n | V | \chi_d \phi_d \rangle|^2 \delta(E - E_p - E_n)
\]
Validity of first order approximation

For $W_{An}$ small, we can apply first order perturbation theory,

$$\frac{d^2\sigma}{d\Omega_p dE_p}(E, \Omega) \quad \text{capture} \approx \frac{1}{\pi} \frac{\langle \phi_n | W_{An} | \phi_n \rangle}{(E_n - E)^2 + \langle \phi_n | W_{An} | \phi_n \rangle^2} \frac{d\sigma_n}{d\Omega}(\Omega) \quad \text{transfer}$$

we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV
Application to surrogate reactions

Surrogate for neutron capture

Desired reaction: neutron induced fission, gamma emission and neutron emission.

The surrogate method consists in producing the same compound nucleus $B^*$ by bombarding a deuteron target with a radio active beam of the nuclear species $A$.

A theoretical reaction formalism that describes the production of all open channels $B^*$ is needed.
Disentangling elastic and non elastic breakup

We show some results for the $^{93}$Nb$(d, p)$ reaction with a 15 MeV deuteron beam (Mastroleo et al., Phys. Rev. C 42 (1990) 683)

- we have used the Koning–Delaroche (Koning and Delaroche, Nucl. Phys. A 713 (2003) 231) optical potential
- the real part of the optical potential has been shifted to reproduce the position of the $L = 3$ resonance
- contributions from elastic and non elastic breakup disentangled.
Obtaining spin distributions

\[ \frac{d\sigma}{dE} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l_p, m} \int |\varphi_{lmlp}(r_{Bn}; k_p)|^2 W(r_{An}) \, dr_{Bn}. \]
Getting rid of Weisskopf–Ewing approximation

- Weisskopf–Ewing approximation:
  \[ P(d, nx) = \sigma(E) G(E, x) \]
- inaccurate for \( x = \gamma \) and for \( x = f \) in the low–energy regime
- can be replaced by \( P(d, nx) = \sum_{J, \pi} \sigma(E, J, \pi) G(E, J, \pi, x) \) if \( \sigma(E, J, \pi) \) can be predicted.
We have presented a reaction formalism for inclusive deuteron–induced reactions.

Valid for final neutron states from Fermi energy to scattering states

Disentangles elastic and non elastic breakup contributions to the proton singles.

Probe of nuclear structure in the continuum.

Provides spin–parity distributions.

Useful for surrogate reactions.

Need for optical potentials.

Extend for \((p, d)\) reactions (hole states)?
The 3–body model

From $H$ to $H_{3B}$

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$
Observables: angular differential cross sections (neutron bound states)

\[ \frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum l,m,l_p \int |\varphi_{lml_p}(r_{Bn}; k_p) Y^{l_p}_{-m}(\theta_p)|^2 W(r_{An}) \, dr_{Bn}. \]

- capture at resonant energies compared with direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor \( \langle \phi_n | W_{An} | \phi_n \rangle \pi. \)
elastıc breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the $U_{An}$ interaction (Koning and Delaroche, Nucl. Phys. A 713 (2003) 231).
Sub-threshold capture

\[ \frac{d\sigma}{dE} \text{ (mb/MeV)} \]

\( W_{\text{An}} = 0.5 \text{ MeV} \)

\( W_{\text{An}} = 3 \text{ MeV} \)

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Non-orthogonality term

$^9$Nb(d,p)

$E_d=25.5$ MeV

$\theta_p=10^\circ$

$\frac{d^2\sigma}{dE d\Omega}$ (mb/MeV sr)

$E_n=-3$ MeV

$E_n=5$ MeV

NEB with non orthogonality

NEB without non orthogonality