



Materials & Related Scorings



Advanced FLUKA Course

Material under irradiation

- The prediction of the structural damage to materials under irradiation is essential to evaluate consequences due to long term employment of construction materials in nuclear reactors and charged particle accelerators.
- There are several effects and quantities that are important for studying the material behavior under irradiation and FLUKA is capable in predicting several of them:
 - Heating total energy deposition
→ could be used for temperature rise, stresses, deformations ...
 - Energy deposition due to EMF → radiolysis calculations ...
 - Dose calculation → medical applications ...
 - Gas production
 - Non Ionizing Energy Losses (NIEL)] → material damage ...
 - Displacement per Atoms (dpa)
 - Silicon 1MeV Neutron Equivalent (Si1MeVEq)] → electronics
 - Single Event Upsets to electronics (SEU)

For all the above you need to input the proper MATERIAL properties

Material definition

Single-element material definition

	atomic number Z	atomic weight	density (g/cm ³)	material number	Alternate material to use for dE/dx	mass number (A)	name
* . . . + . . . 1 . . . + . . . 2 . . . + . . . 3 . . . + . . . 4 . . . + . . . 5 . . . + . . . 6 . . . + . . . 7 . . . + . . .	MATERIAL	24.0	51.9961	7.18	26.0	0.0	0.0 CHROMIUM
MATERIAL		Name: PHOSPHO		#	ρ: 2.2		
Z: 15		Am: 30.973761		A:	dE/dx: ▼		

Notes:

- if $\rho < 0.01$: gas at atmospheric pressure
- Atomic Weight is **calculated by the code** using the internal database is better to leave empty
- Material number use it **ONLY** if you want to **override** a predefined one
- Mass Number to define specific **ISOTOPES**
Do not confuse with the Atomic weight
- Choose a name corresponding to the **LOW-ENERGY neutron database** Section 10.4 in the manual

Predefined ICRU materials

In the new version of FLUKA, the code contains several predefined materials with the composition suggested by ICRU

FLUKA	Material	FLUKA	Material
AIR	Dry air 20°C	BONECOMP	Compact bone
WATER	Water	BONECORT	Cortical bone
POLYSTYR	Polystyrene	MUSCLESK	Skeletal muscle
PMMA	Polymethyl methacrylate	MUSCLEST	Striated muscle
POLYETHY	Polyethylene	ADTISSUE	Adipose tissue
PLASCINT	Plastic scintillator	KAPTON	Kapton

- The materials can be used WITHOUT the need of an explicit MATERIAL / COMPOUND cards
- If the user defines a MATERIAL card in the input with the same name as the predefined ones IT WILL OVERRIDE THE PREDEFINED.

Material Assignment

A (single-element or compound) material is assigned to each geometry region.

*...+...1...+...2...+...3...+...4...+...5...+...6...+...7...+...	MATERIAL	from REGION	to REGION	step	magnetic field	MATERIAL for decay run
ASSIGNMA	GOLD	TARGS1	TARGS3	1.0	0.0	BLCKHOLE
ASSIGNMA	Mat: WATER ▼	Reg: WATERCNT ▼	to Reg: ▼	Step:	Field: ▼	Mat(Decay): BLCKHOLE ▼

In the new version of FLUKA:

- **WHAT(5)** is controlling the magnetic (& electric) field for the prompt and radioactive decay product transport
- **WHAT(6)** is permitting to assign a different material for the radioactive decay product transport.
Only **VACUUM** and **BLCKHOLE** are allowed

MAT-PROP

Card: MAT-PROP

MAT-PROP	Type: ▼	Gas pressure:	RHOR:
Ionization:	Mat: ▼	to Mat: ▼	Step:

- Supply extra information on gaseous materials (pressure)
- Create materials with fictitious or effective density
- Override the default ionization potential

MAT-PROP	Type: DPA-ENER ▼	DPA Eth:	
	Mat: ▼	to Mat: ▼	Step:

- Set the **DPA** energy damage threshold (WARNING in **eV**)

MAT-PROP	Type: USERDIRE ▼	Call: USRMED ▼	
	Mat: ▼	to Mat: ▼	Step:

- Enable the call to **USRMED** routine when a particle enters in the desired materials

There is a last option **LOWNTEMP** to change the material temperature for the new treatment at thermal energies. **HOWEVER** *it cannot be applied with the new library with 260 groups*

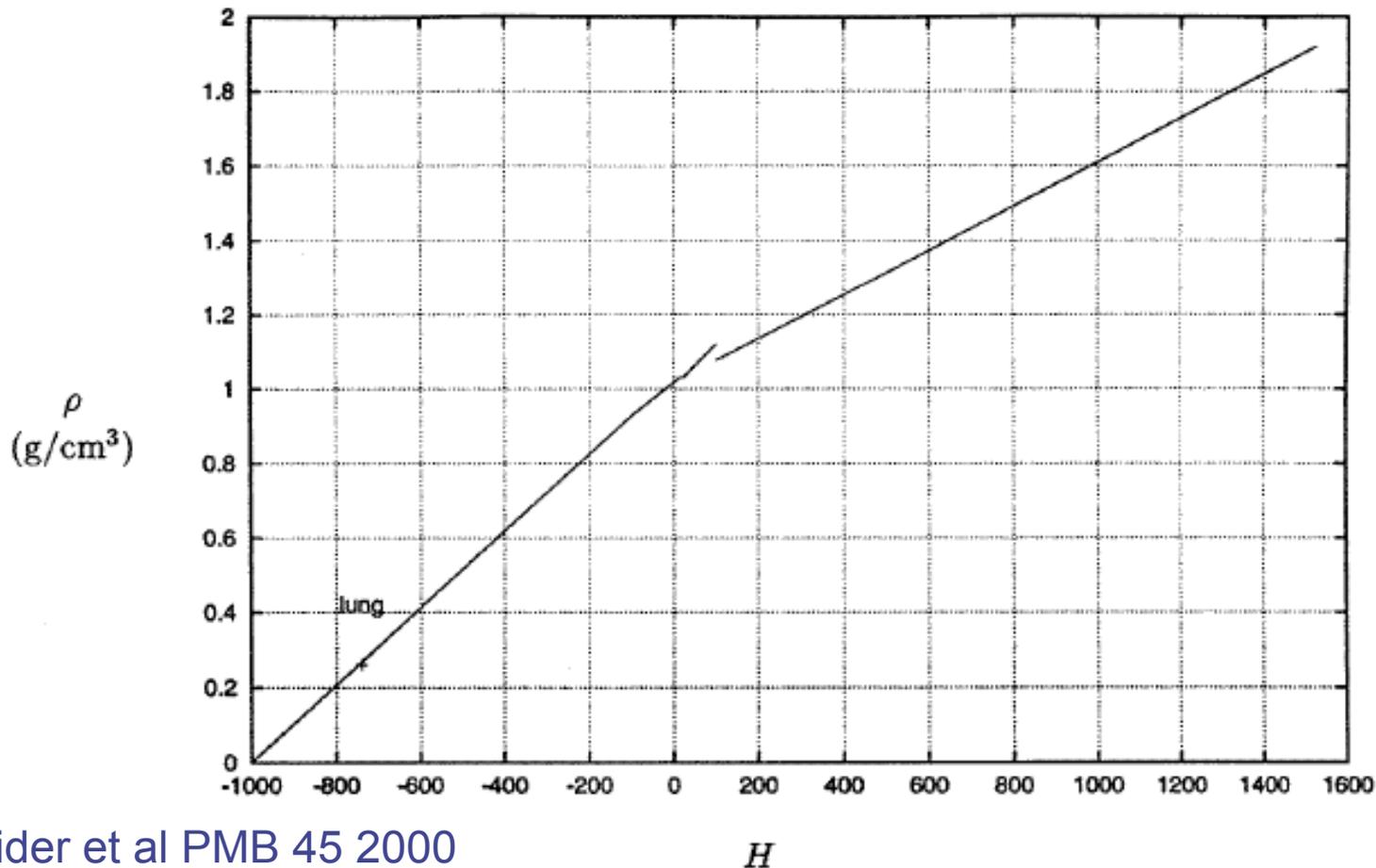
CORRFACT: Region dependent – 1

CORRFACT	xp (dE/dx): Reg: ▼	xp other: to Reg: ▼	Step:
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- CORRFACT card allows to alter material density for dE/dx and nuclear processes
- First two inputs specify a **density scaling factor** (restricted to the interval $[2/3, 3/2]$) for **charged particle ionization processes** (WHAT(1)) and for all other processes (WHAT(2)) to the region(s) specified by the inputs WHAT(4-6)
- This is especially important in ion beam therapy to force the MC to follow the same **semi-empirical Hounsfield Units-range calibration curve** as the Treatment Planning System (TPS) for dosimetric comparisons.
- Typically, a user has to deal with a CT stoichiometric calibration, i.e. a **segmentation of the CT** scan (expressed in Hounsfield units (HU)) into materials of defined elemental composition (for example Schneider *et al* 45 2000).

CORRFAC: Region dependent – 2

- A “nominal density”, e.g., using the density at the center of each HU interval (Jiang et al MP 2004) is assigned to each material. But “real density” (and related physical quantities) varies continuously with HU values.



STERNHEI me card

STERNHEI	Cbar:	X0:	X1:
Mat: ▼	a:	m:	δ0:

- Below the δ -ray threshold, energy losses are treated as “continuous”, with some special features:
 - Fluctuations of energy loss are simulated with a FLUKA-specific algorithm
 - The energy dependence of cross sections and dE/dx is taken into account exactly
 - Latest recommended values of ionization potential and density effect parameters implemented for elements (Sternheimer, Berger & Seltzer), but can be overridden by the user with (set yourself for compounds!) the

STERNHEI C X0 X1 a m δ0 MAT

- In addition, the card MAT-PROP can be used a to override the value of the average ionization potential used by the program

MAT-PROP Gasp Rhosc Iion Mat1 Mat2 Step

dpa: Displacements Per Atom

- Is a measure of the amount of radiation damage in irradiated materials
For example, 3 dpa means each atom in the material has been displaced from its site within the structural lattice of the material an average of 3 times
- Displacement damage can be induced by all particles produced in the hadronic cascade, including high energy photons. The latter, however, have to initiate a reaction producing charged particles, neutrons or ions.
- The dpa quantity is directly related with the total number of defects (or Frenkel pairs)

$$dpa = \frac{1}{\rho} \sum_i N_i N_F^i$$

ρ atoms/cm³

N_i particles per interaction channel i

N_f^i Frenkel pairs per channel

Frenkel pairs

- Frenkel pairs N_F (defect or disorder), is a compound crystallographic defect in which an **interstitial** lies near the **vacancy**. A Frenkel defect forms when an atom or ion leaves its place in the lattice (leaving a vacancy), and lodges nearby in the crystal (becoming an interstitial)

$$N_{NRT} \equiv N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

N_{NRT}
 $\kappa=0.8$
 T

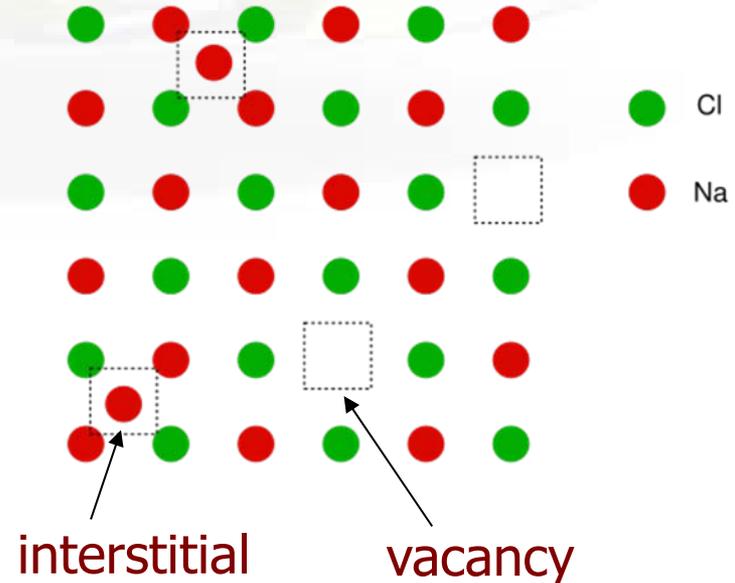
Defects by Norgert, Robinson and Torrens
 is the displacement efficiency
 kinetic energy of the primary
 knock-on atom (PKA)

$\xi(T)$
 $\xi(T) T$

partition function (LSS theory)
 directly related to the **NIEL**
 (non ionizing energy loss)

E_{th}

damage threshold energy



$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

E_{th} Damage Threshold Energy

- E_{th} is the value of the threshold displacement energy averaged over all crystallographic directions or a minimum energy to produce a defect

Element	E_{th} (eV)	Element	E_{th} (eV)
Lithium	10	Co	40
C in SiC	20	Ni	40
Graphite	30..35	Cu	40
Al	27	Nb	40
Si	25	Mo	60
Mn	40	W	90
Fe	40	Pb	25

Typical values used in NJOY99 code

- FLUKA way

MAT-PROP *WHAT(1)* = E_{th} (eV)
 WHAT(4,5,6) = Material range
 SDUM = **DPA-ENER**

FLUKA Implementation [1/2]

Charged particles and heavy ions

• During Interactions

- ✓ ■ For all charged particles and Heavy Ions, calculate the recoil. Use recoil as a normal particle

• During transport

- ✓ ■ Calculate the restricted and unrestricted nuclear stopping power for the average energy at the middle of the step with calls to DEDXNU and SNRDFR and add it to TKNIEL and TKEDPA global variables
- ✗ ■ For electrons Bremsstrahlung, sample randomly a recoil energy from the distribution of the recoils (uncorrelated with the event) Treat the recoil as a normal particle (*work on progress*)

✓ Below threshold

- Calculate the $TKNIEL(=TKEDPA)$ by using the Lindhard partition function

FLUKA Implementation [2/2]

Neutrons

- **High energy $E_n > 20$ MeV**

- ✓
 - Like CP, **calculate the recoil**.
Treat recoil as a normal particle

- **Low energy $E_n \leq 20$ MeV (group-wise)**

- ✓
 - Calculate the NIEL from NJOY, then add it to **TKNIEL** and **TKEDPA** global variables

- **Low energy $E_n \leq 20$ MeV (point-wise)**

- ~~X~~
 - Calculate the recoil if possible
- ✓
 - Treat the recoil as a normal particle
(available for those where point-wise exist)

Photons

- ~~X~~
 - **Pair production**, sample randomly a recoil for a distribution
(uncorrelated with the event) Treat the recoil as a normal particle

dpa: Recipe

FLUKA is using a more accurate treatment during the particle transport. While below the transport threshold is employing the Lindhard approximation with the NRT model, it is strongly advisable to use as low thresholds as possible:

Thresholds:

- All Hadrons 1 keV
- Neutrons down to thermal ($1e-15$ GeV)
- Leptons 50-100 keV would be ok

Material Damage:

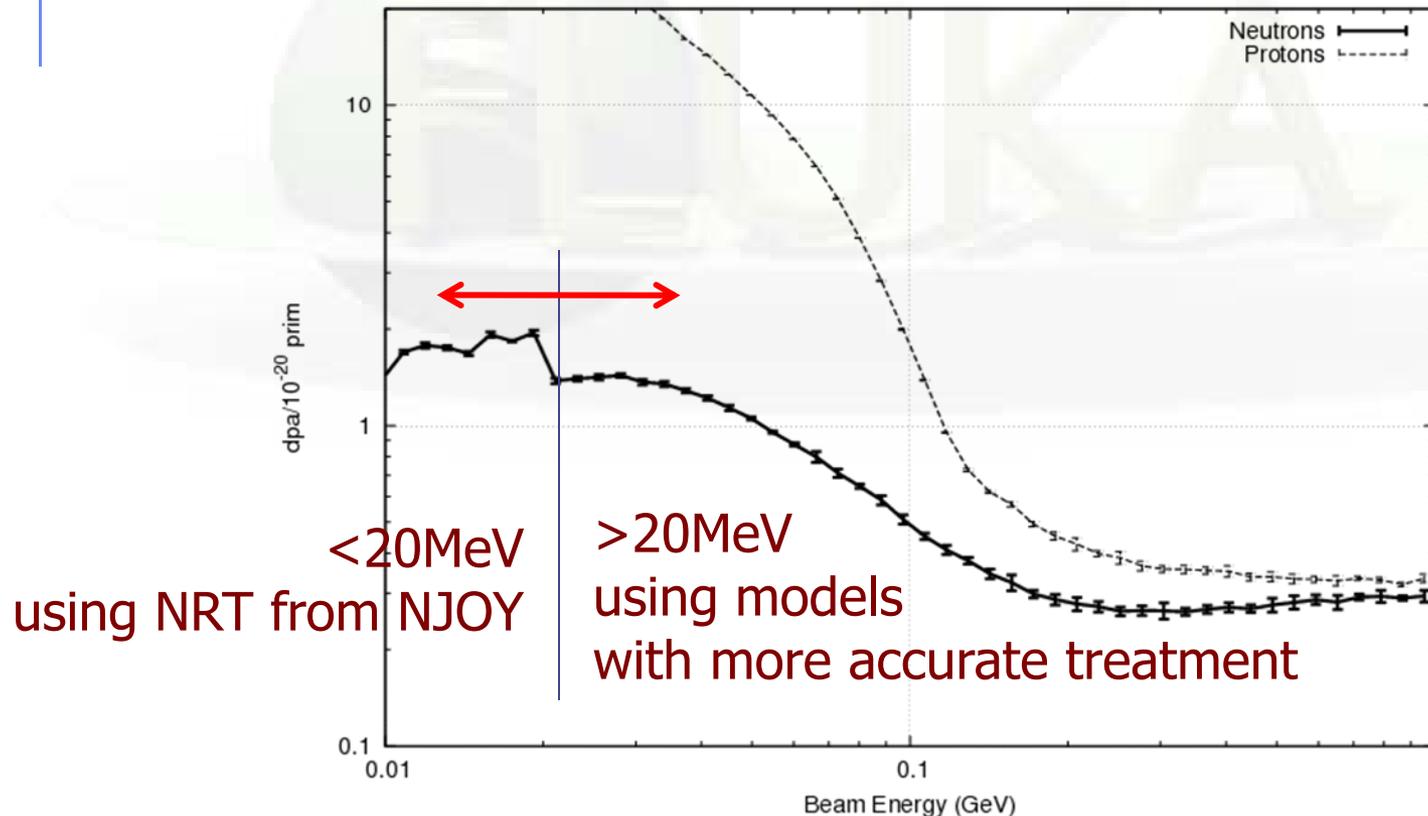
Set with **MAT-PROP** the damage threshold for all materials under consideration. All other will use the default of **30eV!!!!**

Scoring:

Use **USRBIN** with **DPA-SCO** or **NIEL**

dpa: Artifacts

- Due to the group treatment of low-energy neutrons, there is no direct way to calculate properly the recoils.
- Therefore the evaluation is based on the KERMA factors calculated by NJOY, which in turn is based on the Unrestricted Nuclear losses from using the NRT model.



Cards Displacement Damage + Charge

For all charged particles and Heavy Ions FLUKA calculates the recoil as a normal particle. During transport it calculates the restricted and unrestricted nuclear stopping power, allowing to score dpa's and non-ionizing energy loss (NIEL):

NIEL-DEP Non Ionizing Energy Loss deposition
DPA-SCO Displacements per atoms

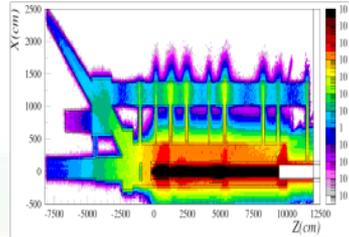
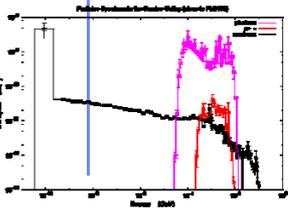
In addition (not necessarily linked to displacement damage) the following can be useful in order to get the net charge deposition in a given region:

NET-CHRG Net Charge

Radiation Physics/Effects/Monitoring

nuclear cascade

$h > 20 \text{ MeV}$



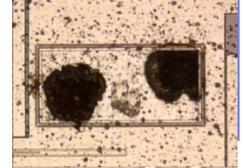
Radiation Field

$h, e, \dots > 100 \text{ KeV}$

EM cascade

radiation damage in semiconductors

Single Events



Effect in the Device

Dose

Displacement

radiation monitor



Radfet

SEU counter

Measurement

PIN Diodes



Main Radiation Effects on Electronics

Category		Effect
Single Event effects (Random in time)	<i>Single Event Upset (SEU)</i>	Memory bit flip (soft error) Temporary functional failure
	<i>Single Event Latchup (SEL)</i>	Abnormal high current state Permanent/destructive if not protected
Cumulative effects (Long term)	<i>Total Ionizing Dose (TID)</i>	Charge build-up in oxide Threshold shift & increased leakage current Ultimately destructive
	<i>Displacement damage</i>	Atomic displacements Degradation over time Ultimately destructive

Radiation Damage to Electronics

- All important quantities to estimate risks of damage to electronics can be directly scored in FLUKA :

Cumulative damage:

- Energy deposition (total ionizing dose) by scoring **DOSE** with any 'energy deposition like estimator' (*e.g.*, **USRBIN**)
- Si Lattice displacement (1-MeV neutron equivalent particle fluxes) with any 'fluence like estimator' (*e.g.*, **USRTRACK**)

Stochastic failures (SEU):

- "high" energy hadron fluences ("E>20 MeV") with any 'fluence like estimator' (*e.g.*, **USRTRACK**)
(+the option of special threshold functions together with the scoring related to the "damage by thermal neutrons")
- The powerful FLUKA scoring options together with the analysis of particle energy spectra allows a detailed study in order to select best possible locations for electronics or efficiently design shielding implementations

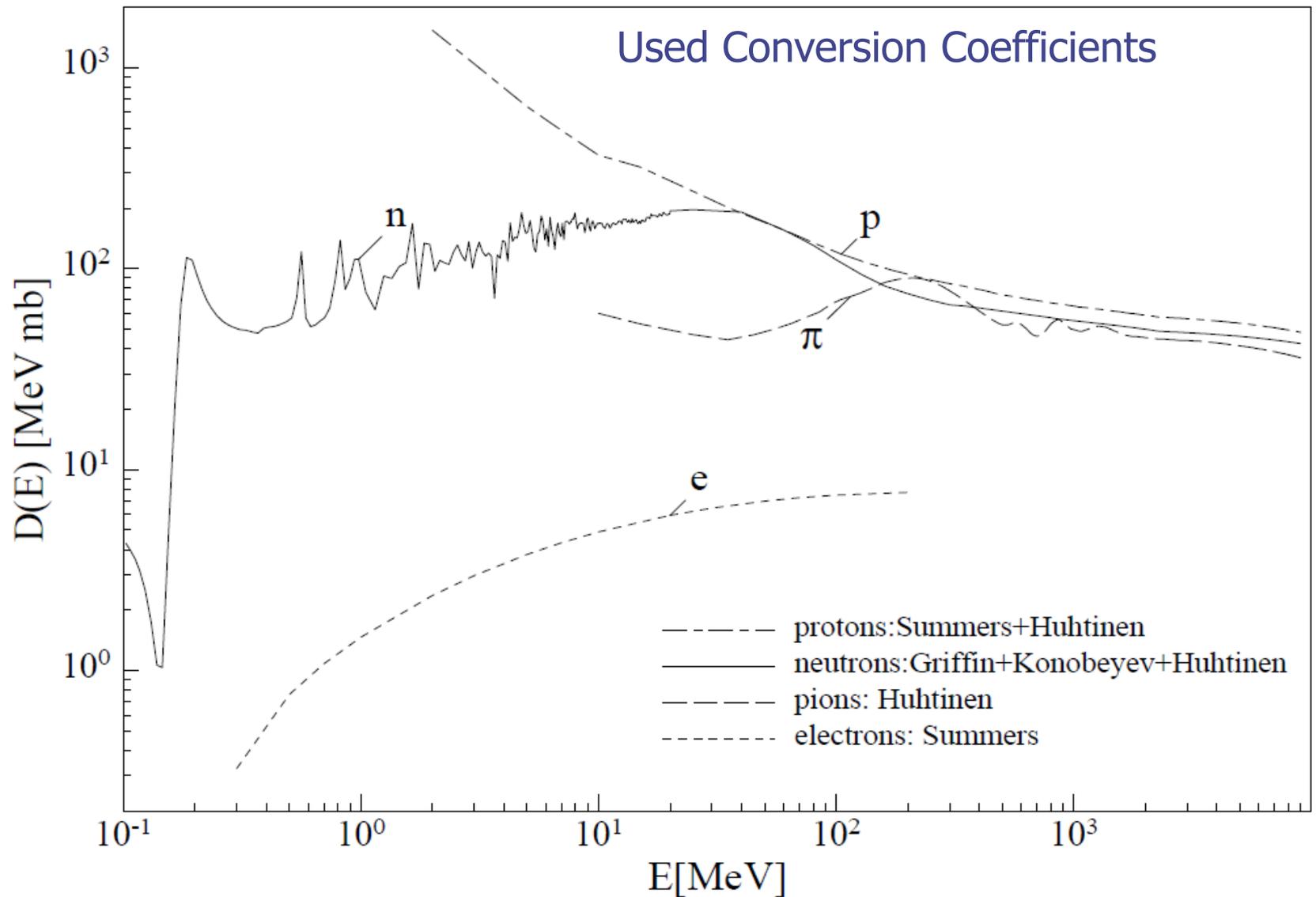
Corresponding FLUKA Estimators

Category		Scales with simulated/measured quantity
Single Event effects (Random in time)	Single Event Upset (SEU) *	HADGT20M [cm ⁻²] (+/or HEHAD-EQ, THNEU-EQ)
	Single Event Latchup (SEL) **	HADGT20M [cm ⁻²] (+/or HEHAD-EQ)
Cumulative effects (Long term)	Total Ionizing Dose (TID)	DOSE [GeV/g] -> stricly IONIZING only!
	Displacement damage	SI1MEVNE [cm ⁻²] {NIEL}

* Reality is often complicated (*e.g.*, contribution of thermal neutrons)

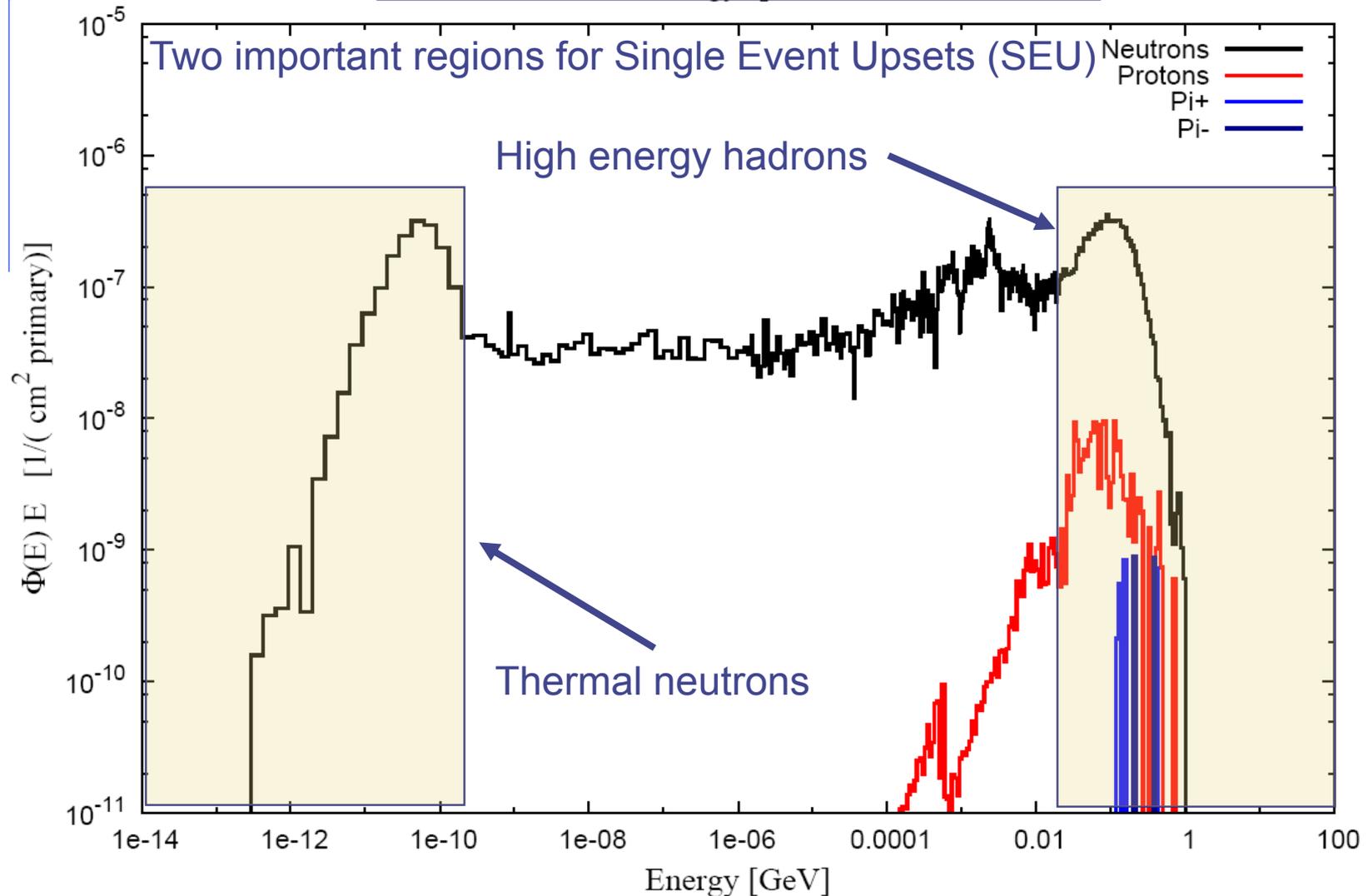
** Energy threshold for inducing SEL is often higher than 20 MeV

1MeV Neutron Equivalent



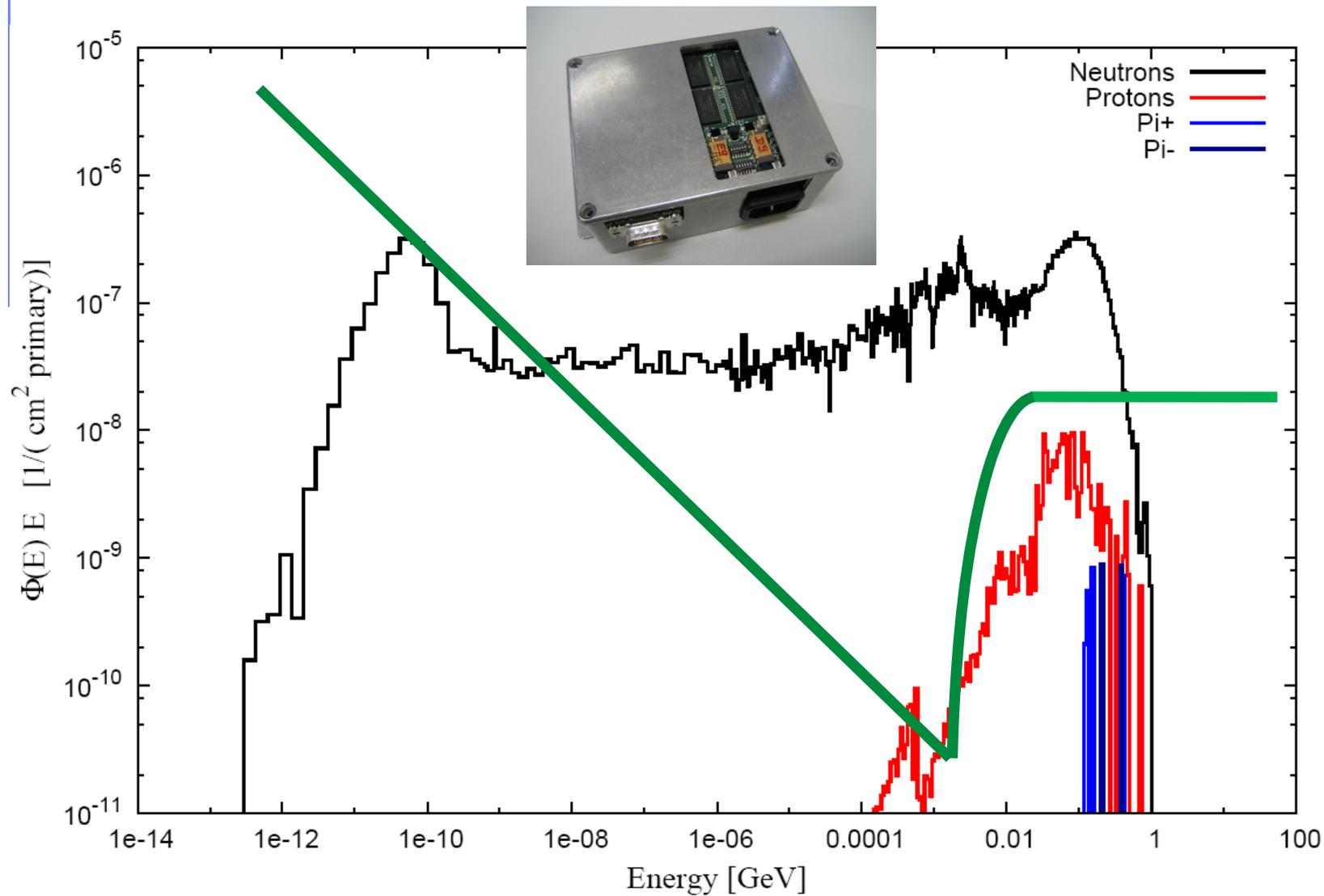
SEUs in mixed radiation field

$$\#SEU = \sigma_{Th. n.} \cdot \Phi_{Th. n.} + \sigma_{HEH} \cdot \Phi_{HEH}$$

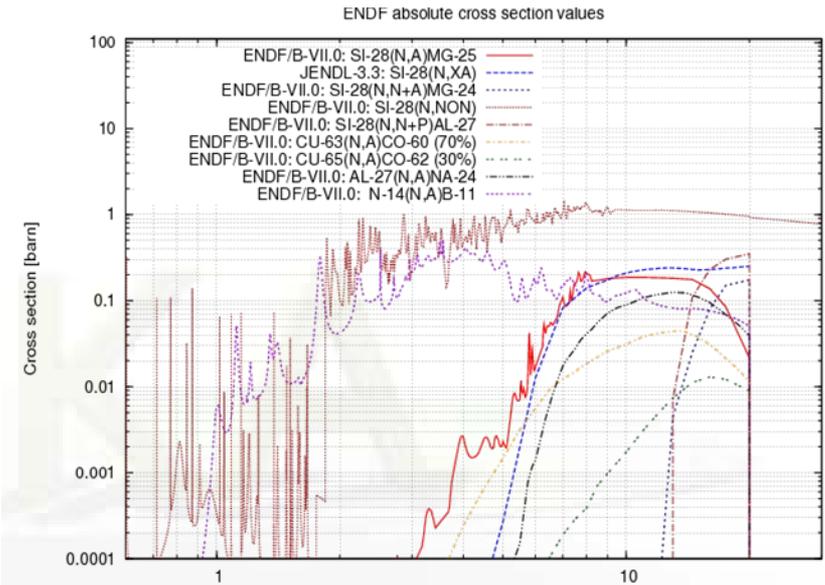
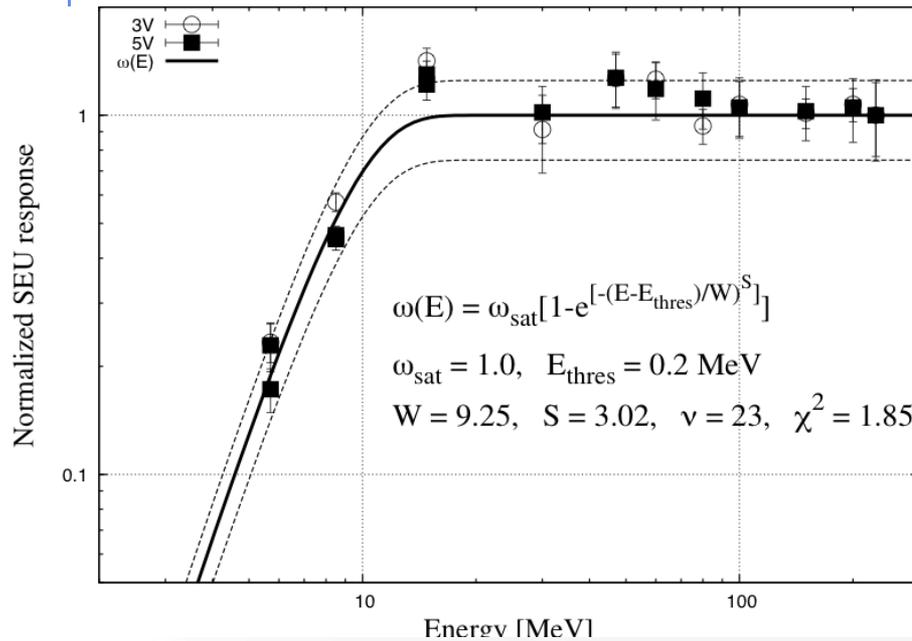


Device response

e.g. LHC RadMon



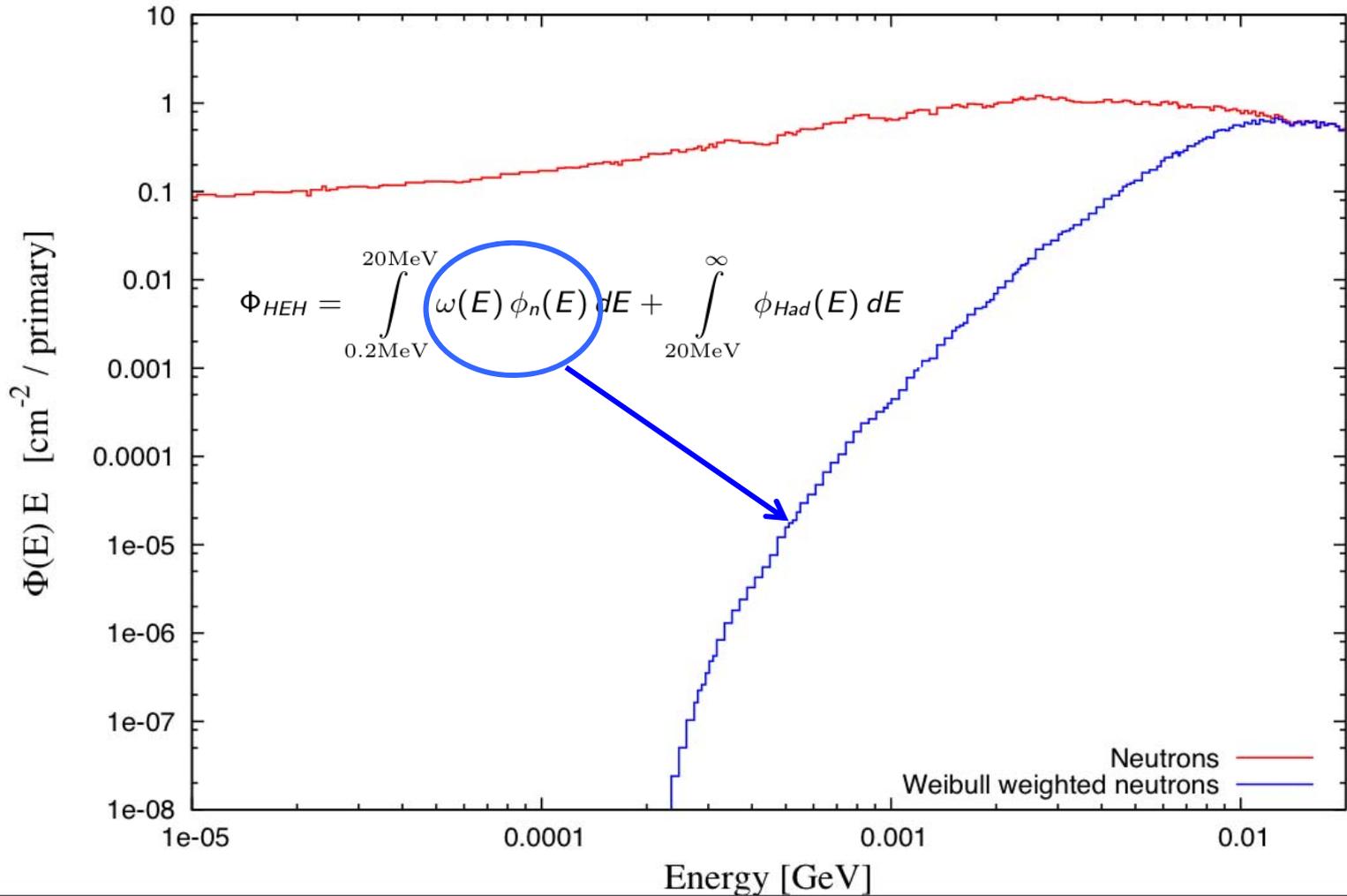
HEHAD-EQ



$$\Phi_{HEH} = \int_{0.2\text{MeV}}^{20\text{MeV}} \omega(E) \phi_n(E) dE + \int_{20\text{MeV}}^{\infty} \phi_{Had}(E) dE$$

HEHAD-EQ

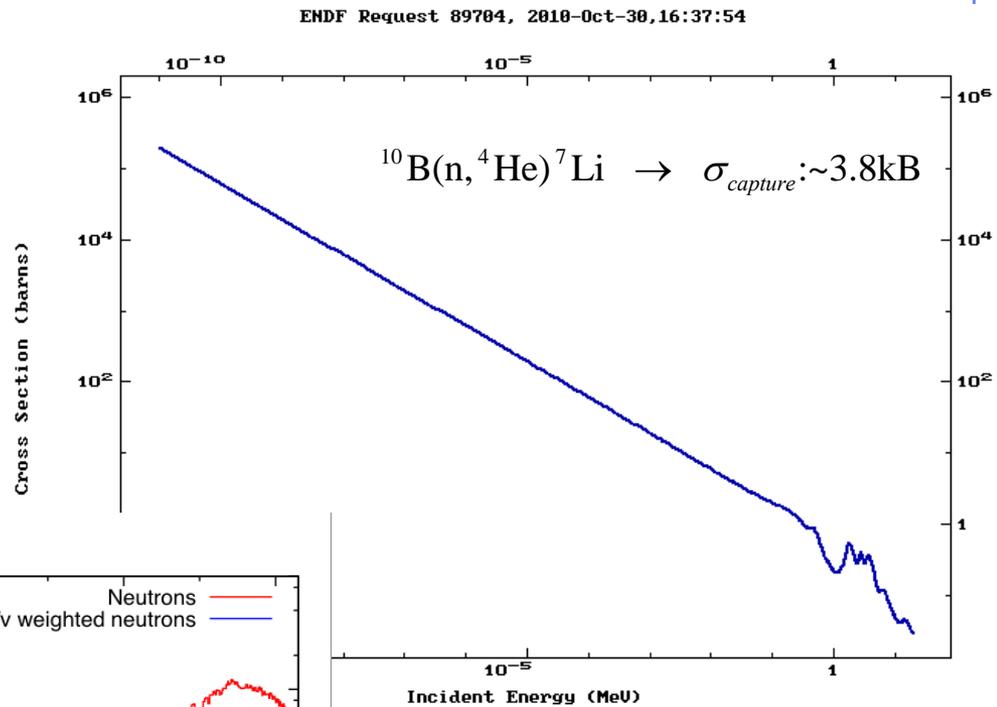
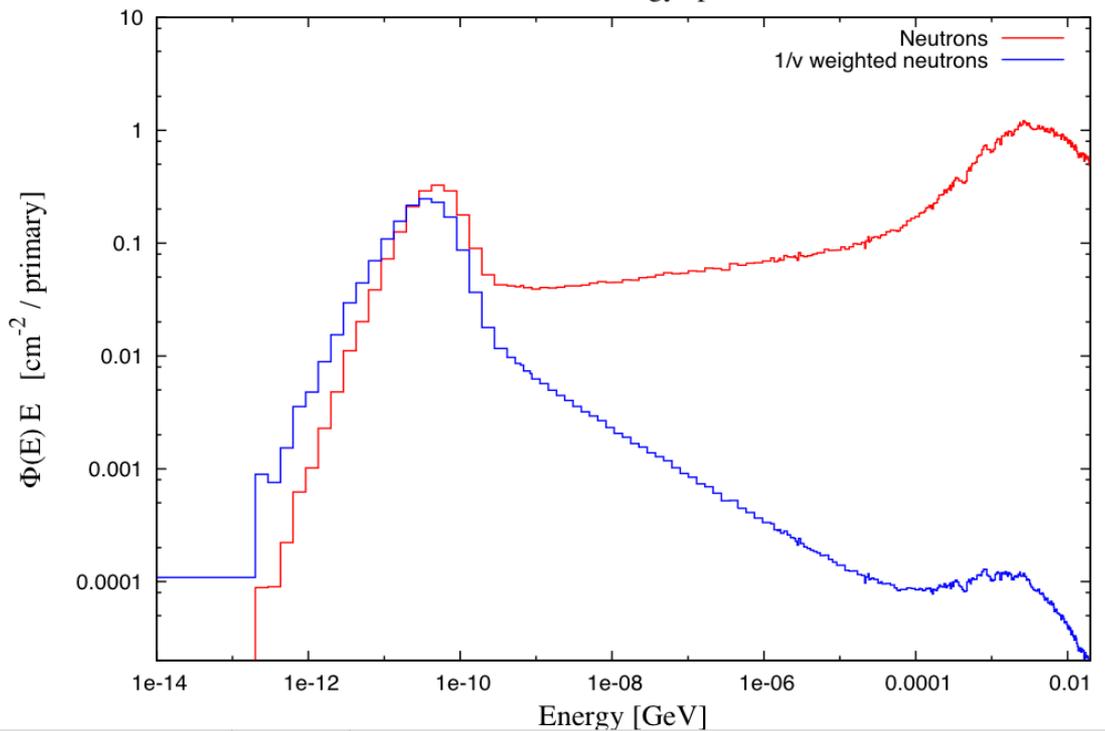
Neutron lethargy spectra



THNEU-EQ



Neutron lethargy spectra



Related Scoring CARDS & Quantities

DOSE	total 'ionizing' dose (TID) in (obviously...)	GeV/g!
SI1MEVNE	Silicon 1 MeV-neutron equivalent fluence	
HADGT20M	Hadrons fluence with energy > 20 MeV	
HEHAD-EQ	as above, but weighted for n < 20 MeV	
THNEU-EQ	1/v weighted neutrons	

- **USRTRACK** scores average $d\Phi/dE$ (differential fluence) in a given region (SI1MEVNE, HADGT20M or any particle type)
- **USRBDX** scores for the same quantities average $d^2\Phi/dE d\Omega$ (double-differential fluence or current) on a given surface (between two regions)
- **USRBIN** scores the spatial distribution either of deposited dose, or fluence (1MeV or 20MeV) in a regular mesh (cylindrical or Cartesian) described by the user
- **USRBIN** also scores the same quantities on a region basis

* 1) high-energy hadron fluence spectrum

```
USRTRACK      -1.  HADGT20M      -31.  RADMON1      125.  170.Ust20MeV
USRTRACK      1D3      1D-14
&
```

* 2) displacement damage spectrum

```
USRBDX        98.  SI1MEVNE      -41.  TAIR  RADMON1  150.Usx1MeV
USRBDX        1D3      1D-14      170.  &
```

* 3) dose distribution in a regular mesh through the geometry

```
USRBIN        10.  DOSE          -21.  100.  20.  200.UsbDose
USRBIN       -100.  -20.  -100.  100.  20.  150.&
```

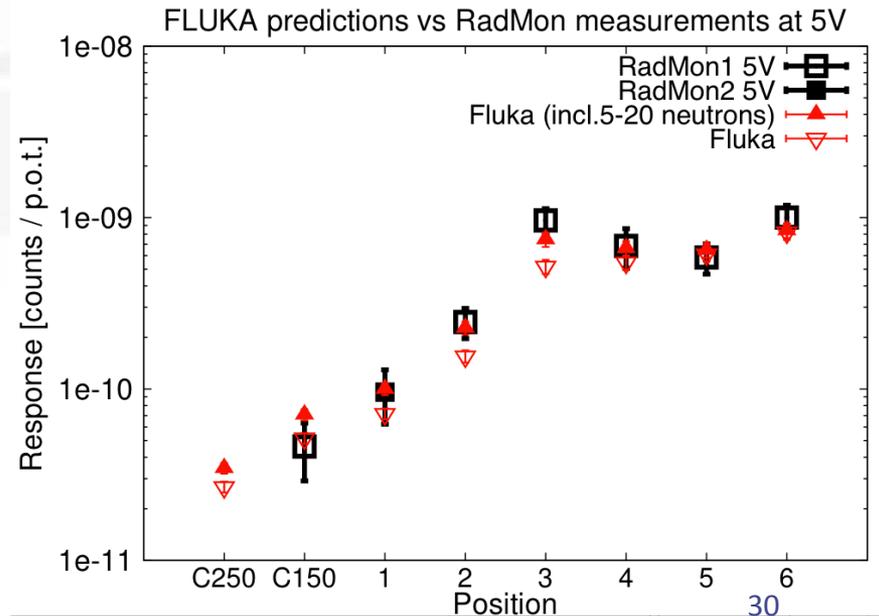
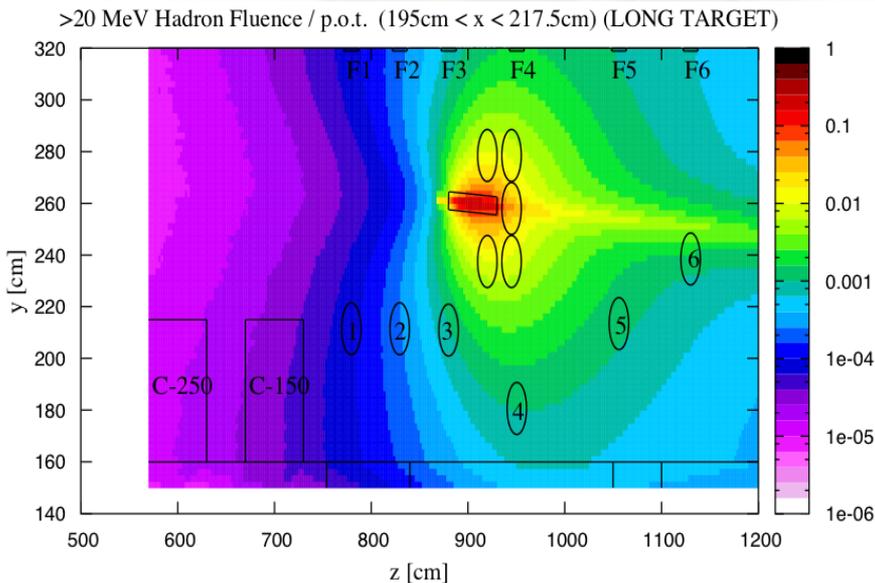
* 4) integrated high-energy hadron equivalent fluence on a region basis

```
USRBIN        18.0  HEHAD-EQ      -37.0  LSTREG  300.0  10000.0UrbReg20
USRBIN        FSTREG  0.0  -10000.0  1.0  1.0  1.0  &
```

Application benchmark @ CERF

- Response of monitor measured in a mixed field facility and compared to predictions by **Monte Carlo simulations**

$$\#SEU = \sigma_{Th. n.} \cdot \Phi_{Th. n.} + \sigma_{HEH} \cdot \Phi_{HEH}$$





Neutrons

Reminder Neutrons

- In FLUKA we call neutrons below 20 MeV **low energy neutrons**
- Neutron interactions at higher energy are handled by FLUKA nuclear models
- Transport and interactions of neutrons with energies below 20 MeV are handled by a dedicated library

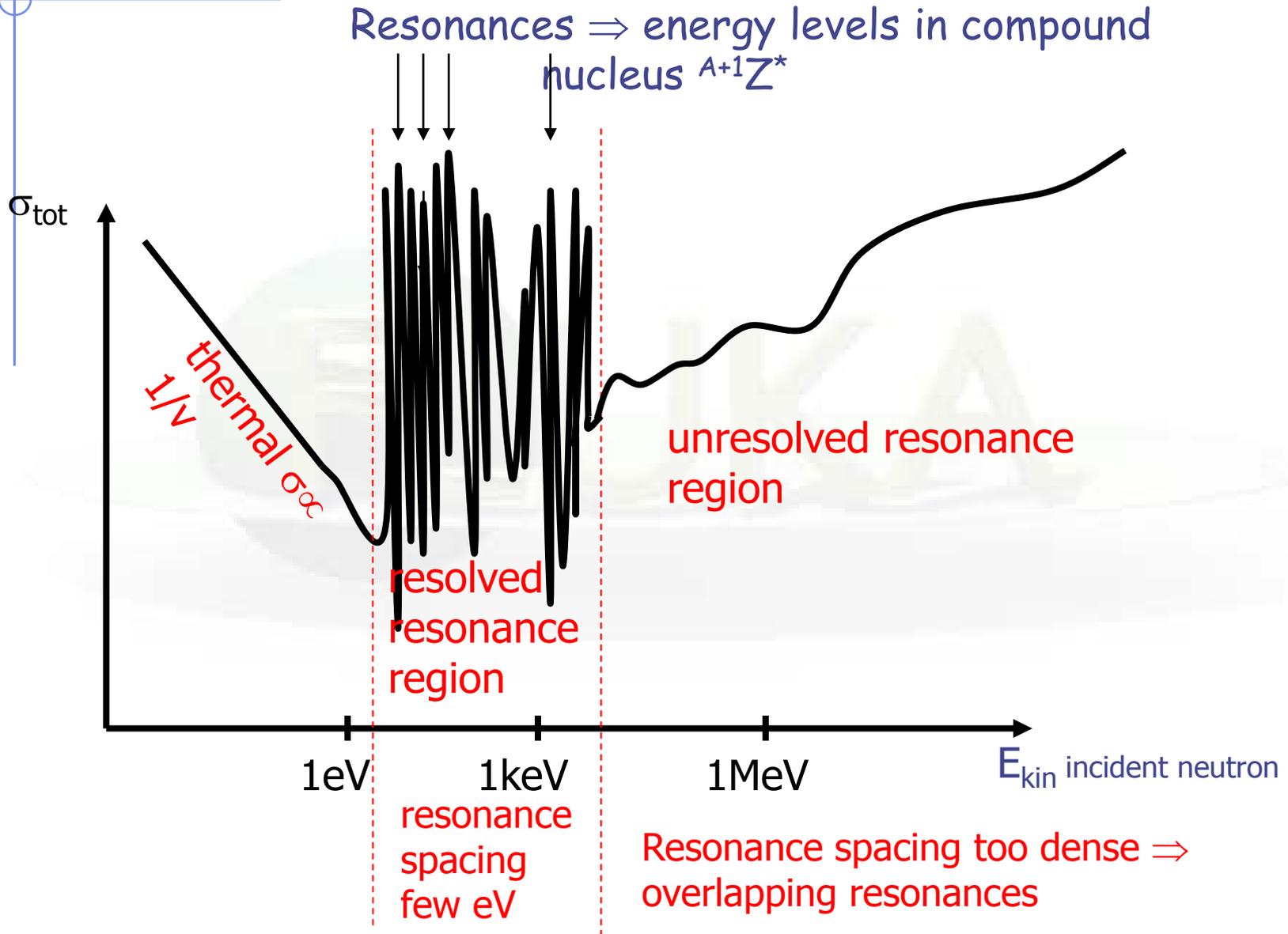
Why are low Energy Neutrons special?

- The neutron has no charge → can interact with nuclei at low energies, e.g. meV
- Neutron cross sections (σ) are complicated → cannot be calculated by models → we rely on data files

Material Correspondance: LOW-MAT

- The **LOW-MAT** card sets the correspondence between FLUKA materials and the low energy neutron cross sections
- If a material has the same name as a name given in the list of low neutron materials, the correspondence between material and low energy neutron transport is set automatically, and a **LOW-MAT** card is not necessary. The **first material** with the right name is taken. This is always a material at room temperature.
- That means that **for the predefined material HYDROGEN hydrogen bound in water is used**, not the free gas one
- If you want to use low energy neutron transport in H₂ gas you have to do this explicitly by a **LOW-MAT** card

Typical neutron cross section



Basis: Evaluated Nuclear Data Files

- Evaluated nuclear data files (ENDF, JEFF, JENDL...)
 - typically provide neutron σ (cross sections) for $E < 20\text{MeV}$ for all channels
 - σ are stored as continuum + resonance parameters

Point-wise and Group-wise cross sections

- In neutron transport codes in general two approaches are used: **point-wise** ("continuous" cross sections) and **group-wise** transport
- Point-wise follows cross section precisely but it can be time and memory consuming
- Group approach is widely used in neutron transport codes because it is fast and gives good results for most application

Complex programs (NJOY, PREPRO...) convert ENDF files to **point-wise or group-wise cross sections**, including Doppler broadening etc.

FLUKA: Point-Wise Neutron Cross Sections

- ***Point-wise neutron transport*** is available for ^1H (above 10 eV if bound H requested, down to 10^{-5} eV otherwise) and ^6Li
- ***Detailed correlated reaction products*** are available for ^1H , ^6Li , ^{10}B (only for the reaction $^{10}\text{B}(n,\alpha)^7\text{Li}$), and the $^{14}\text{N}(n,p)$ reaction. All reaction products are then transported explicitly according to transport setting (**PHYSICS**).
- Recoil proton production is ON by default for H and $^{14}\text{N}(n,p)$
- while for the others and for point-wise treatment it depends on the **DEFAULT** set chosen
- Both are important for precision studies, detector response (exp. scintillators), borated materials...
- To require ***point-wise neutron transport and reaction products*** (where available), use the **LOW-NEUT** card with $\text{WHAT}(6)=1$.

Materials with molecular binding

- Available materials with molecular bindings at 296K:
 - H (natural isotopic amount) in H_2O , CH_2
 - ^1H in H_2O , CH_2
 - ^2D in D_2O
 - C in graphite
- Use of these materials makes the thermal neutron calculation more realistic and can affect the energy and spatial distributions
- Example: CH_2 (polyethylene) including molecular binding
 - Create a material hydrogen and give a corresponding **LOW-MAT** card that refers to H bound in CH_2
 - Give a **COMPOUND** card that creates CH_2 as a compound of bound H and normal carbon
- Reminder: for hydrogen, **H bound in water is the default**, because it is the first in the list of low energy neutron materials

Self-shielding [1/4]

- The group structure is necessarily coarse with respect to the **resonance structure** in many materials
- A resonance in a material present in a dilute mixture or as a small piece cannot affect much a smooth neutron flux (so-called "infinite dilution")
- But if an isotope exhibiting large resonances is **very pure** or is present with a **large fractional abundance**, it can act as a "neutron sink", causing sharp dips in the neutron spectrum corresponding to each resonance → an apparent decrease in σ
- This effect, which results in a lower reaction rate $\sigma\Phi$, is called **self-shielding** and is necessarily **lost in the process of cross section averaging** over the width of each energy group, unless a special correction is made

Self-shielding [2/4]

- Self-shielded materials in FLUKA:

- ^{27}Al at 296K, 87K, 4K, 430K

- $\text{natA, } ^{40}\text{Ar}$ at 296K, 87K

- natFe at 296K, 87K, 4K, 430K

- natCu at 296K, 87K, 4K, 430K

- ^{181}Ta at 296K, 87K

- natW at 296K, 87K, 4K, 430K

- ^{197}Au at 296K, 87K

- natPb at 296K, 87K

- ^{208}Pb at 296K

- ^{209}Bi at 296K, 87K

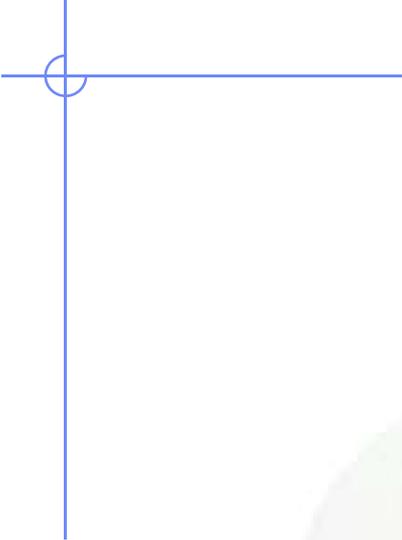
- ◆ Special case: **cast iron** ($\text{natFe} + 5\%\text{C}$) at 296K, 87K, 4K, 430K (see slide further on)

Self-shielding [3/4]

- When to use these materials?
 - Bulky (huge) pieces that are very pure (containing only one isotope)
- When not to use self-shielded materials?
 - “small” iron, copper, lead, aluminum pieces
 - Thin gold foils (but a self-shielded 100 μ m Au foil is available)
 - Diluted materials
- How to use self-shielded materials?
 - Define your material with a **MATERIAL** card
 - Give additionally a **LOW-MAT** card and give the proper identifiers in WHAT(2)-WHAT(4) and SDUM
 - If you have to use self-shielded and non self-shielded materials of the same element you need to define 2 different materials
 - Attention: predefined materials like iron, copper and lead are not self-shielded, you have to give a **LOW-MAT** card to use them self-shielded

Self-shielding [4/4]

- **Cast iron** is iron with a significant amount of carbon
- There is a self-shielded material cast iron in the low energy neutron library which is prepared to be used for creating a compound of iron and roughly 5% carbon. The amount of carbon doesn't need to be exactly 5%.
- **How to create self-shielded cast iron?**
 - Define a material iron called FeCarbSS (or any other name you like) with a **MATERIAL** card (parameters as for natural iron)
 - Insert a **LOW-MAT** card for FeCarbSS with the proper **identifiers for cast iron** in WHAT(2)-WHAT(4) and SDUM
 - Insert a **MATERIAL** card to declare a compound material called CastFe (or any other name you like)
 - Insert a **COMPOUND** card for defining CastFe as a compound of FeCarbSS and CARBON (predefined)



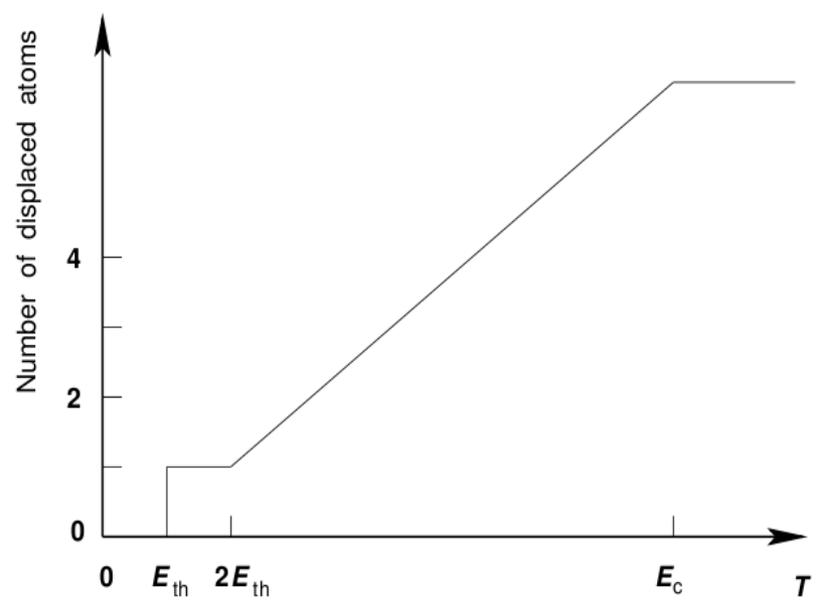
Backup

$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

Factor of 2 (Kinchin & Pease)

- The cascade is created by a sequence of two-body elastic collisions between atoms
- In the collision process, the energy transferred to the lattice is zero
- For all energies $T < E_c$ electronic stopping is ignored and only atomic collisions take place. No additional displacement occur above the cut-off energy E_c
- The energy transfer cross section is given by the **hard-sphere** model.

$v(T)=0$	for $0 < T < E_{th}$ (phonons)
$v(T)=1$	for $E_{th} < T < 2E_{th}$
$v(T)=T/2E_{th}$	for $2E_{th} < T < E_c$
$v(T)=E_c/2E_{th}$	for $T > E_c$



Schematic relation between the number of displaced atoms in the cascade and the kinetic energy T of the primary knock-on atom

Energy is equally shared between two atoms after the first collision
Compensates for the energy lost to sub threshold reactions

$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

κ displacement efficiency

- $\kappa=0.8$ value deviates from the **hard sphere model** (K&P), and compensates for the forward scattering in the displacement cascade
- The displacement efficiency κ can be considered as independent of T only in the range of $T \leq 1-2$ keV. At higher energies, the development of collision cascades results in **defect migration** and **recombination of Frenkel pairs** due to overlapping of different branches of a cascade which translates into decay of $\kappa(T)$.
- From molecular dynamics (MD) simulations of the primary cascade the number of surviving displacements, N_{MD} , normalized to the number of those from NRT model, N_{NRT} , decreases down to the values about 0.2–0.3 at $T \approx 20-100$ keV. The efficiency in question only slightly depends on atomic number Z and the temperature.

$$N_{MD}/N_{NRT} = 0.3-1.3$$

$$N_{MD} / N_{NRT} = 0.3 - 1.3 \left(-\frac{9.57}{X} + \frac{17.1}{X^{4/3}} - \frac{8.81}{X^{5/3}} \right)$$

where $X \equiv 20 T$ (in keV).

$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

Lindhard partition function ξ [1/3]

- The partition function gives the fraction of **stopping power** S that goes to NIEL

$$\xi(T) = \frac{S_n}{S} \quad \text{where} \quad S = \frac{dE}{dx} = \frac{dE_n}{dx} + \frac{dE_e}{dx} = S_n + S_e$$

- Lindhard, Robinson assumption:

The incident particle has energy E , and due to the interactions during the slowing down the energy is transferred to atoms E_n and to electrons E_e

$$E = E_e + E_n$$

The equation that has to be satisfied is

$$\int d\sigma_{n,e} \left[E_n \left(E - T_n - \sum_i T_{ei} \right) - E_n(E) + E_n(T_n - U) + \sum_i E_{ne}(T_{ei} - I_i) \right] = 0$$

where $d\sigma_{n,e}$ are the differential cross-sections corresponding to particle scattering on nucleus and electrons, T_n (T_{ei}) energy transfer to nucleus (electrons), U energy in atomic(lattice) binding, I_i ionization energies

$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$

Lindhard partition function ξ [2/3]

- Approximations used: Electrons do not produce recoil nuclei with appreciable energy, lattice binding energy is neglected, etc...

$$(S_n + S_e)E'_n(E) = \int E_n(T) \frac{d\sigma_n}{dT} dT$$

where

$$S_{n,e}(E) = \int T_{n,e} d\sigma_{n,e}$$

- approximated to

$$\xi(T) = \frac{1}{1 + F_L \cdot (3.4008 \cdot \varepsilon(T)^{1/6} + 0.40244 \cdot \varepsilon(T)^{3/4} + \varepsilon(T))}$$

$$F_L = 30.724 \cdot Z_1 \cdot Z_2 \sqrt{Z_1^{2/3} + Z_2^{2/3}}$$

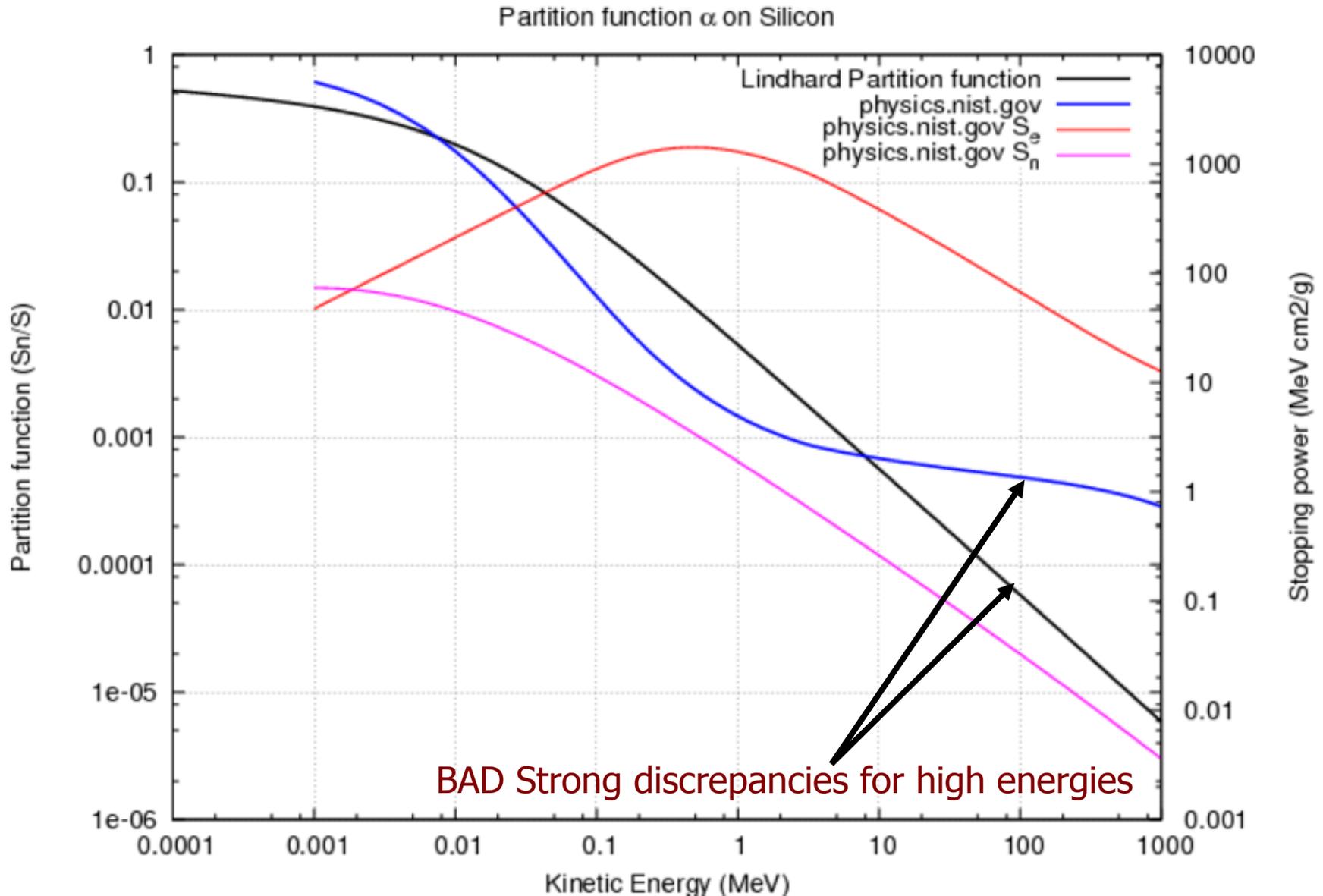
$$\varepsilon(T) = \frac{T}{0.0793 \frac{Z_1^{2/3} \cdot \sqrt{Z_2}}{(Z_1^{2/3} + Z_2^{2/3})^{3/4}} \cdot \frac{(A_1 + A_2)^{3/2}}{A_1^{3/2} \sqrt{A_2}}}$$

Z,A	charge and mass
1	projectile
2	medium
T	recoil energy (eV)

Nice feature: It can handle any projectile Z_1, A_1 whichever charged particle

Lindhard partition function ξ [3/3]

$$N_F = \kappa \frac{\xi(T)T}{2E_{th}}$$



Restricted Nuclear Stopping Power

- Lindhard approximation uses the **unrestricted NIEL**. Including all the energy losses also those below the threshold E_{th}
- FLUKA is using a more accurate way by employing the **restricted nuclear losses**

$$S(E, E_{th}) = N \int_{E_{th}}^{\gamma E} T \left(\frac{d\sigma}{dT} \right) dT$$

where:

$S(E, E_{th})$ is the restricted energy loss

N atomic density

T energy transfer during ion-solid interaction

$d\sigma/dT$ differential scattering cross section

$\gamma = \frac{4M_1M_2}{(M_1 + M_2)^2}$ maximum fraction of energy transfer during collision

App. benchmark: IR7 collimation region