



# Hadron-Nucleus Interactions

Beginners' FLUKA Course

# The FLUKA hadronic Models

Hadron-nucleus: PEANUT

Elastic, exchange  
Phase shifts  
data, eikonal

$P < 3-5 \text{ GeV}/c$   
Resonance prod  
and decay

Sophisticated  
G-Intranuclear Cascade

Gradual onset of  
Glauber-Gribov multiple  
interactions

Preequilibrium

Coalescence

hadron

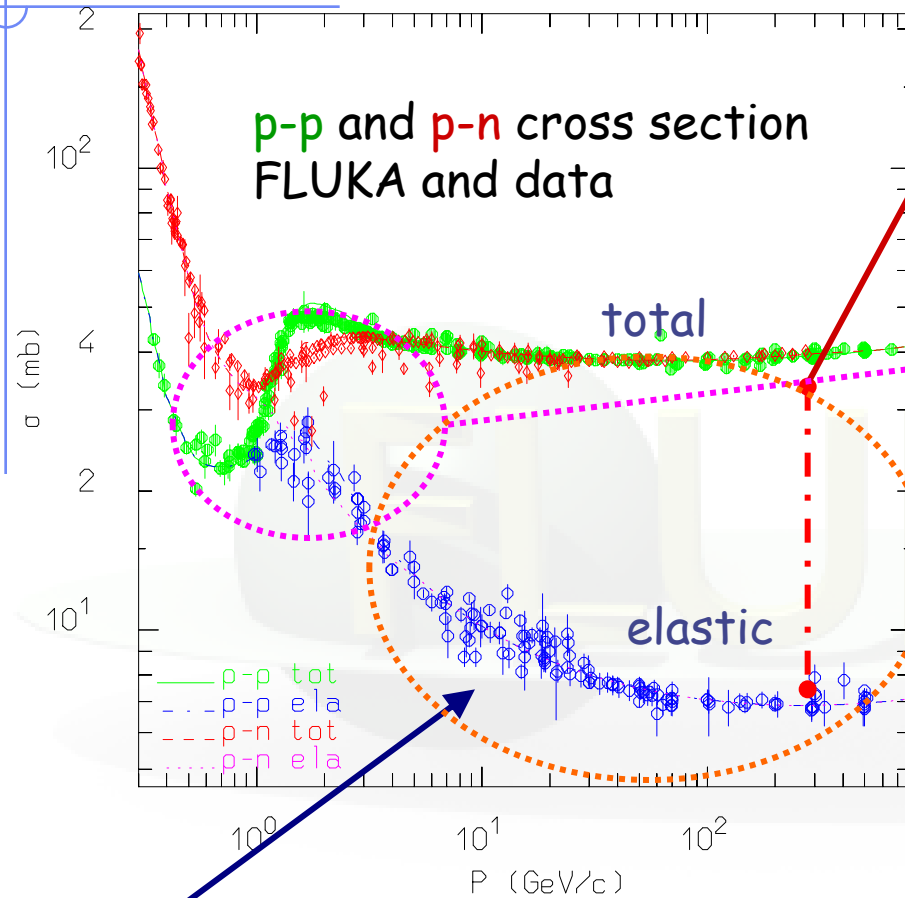
hadron

low E  
 $\pi, K$   
Special

High Energy  
DPM  
hadronization

Evaporation/Fission/Fermi break-up  
 $\gamma$  deexcitation

# Hadron-nucleon interaction models



Particle production interactions:  
two kinds of models

Those based on "resonance"  
production and decays, cover the  
energy range up to 3-5 GeV

Those based on quark/parton  
string models, which provide  
reliable results up to several tens  
of TeV

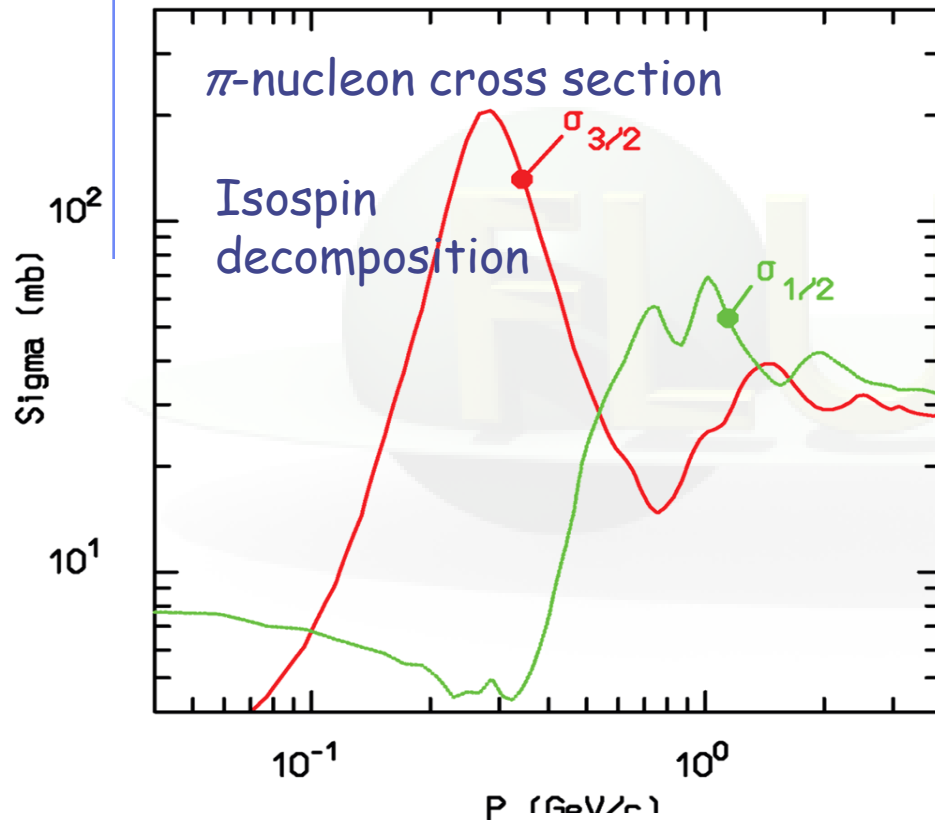
Elastic, charge exchange and strangeness exchange reactions:

- Available phase-shift analysis and/or fits of experimental differential data
- At high energies, standard eikonal approximations are used

# Nonelastic hN interactions at intermediate energies

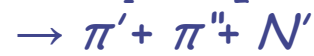
- $N_1 + N_2 \rightarrow N'_1 + N'_2 + \pi$  threshold at 290 MeV, important above 700 MeV,
- $\pi + N \rightarrow \pi' + \pi'' + N'$  opens at 170 MeV.

Anti-nucleon-nucleon open at rest !



Dominance of the  $\Delta$  resonance and of the  $N^*$  resonances  
 → isobar model  
 → all reactions proceed through an intermediate state containing at least one resonance.

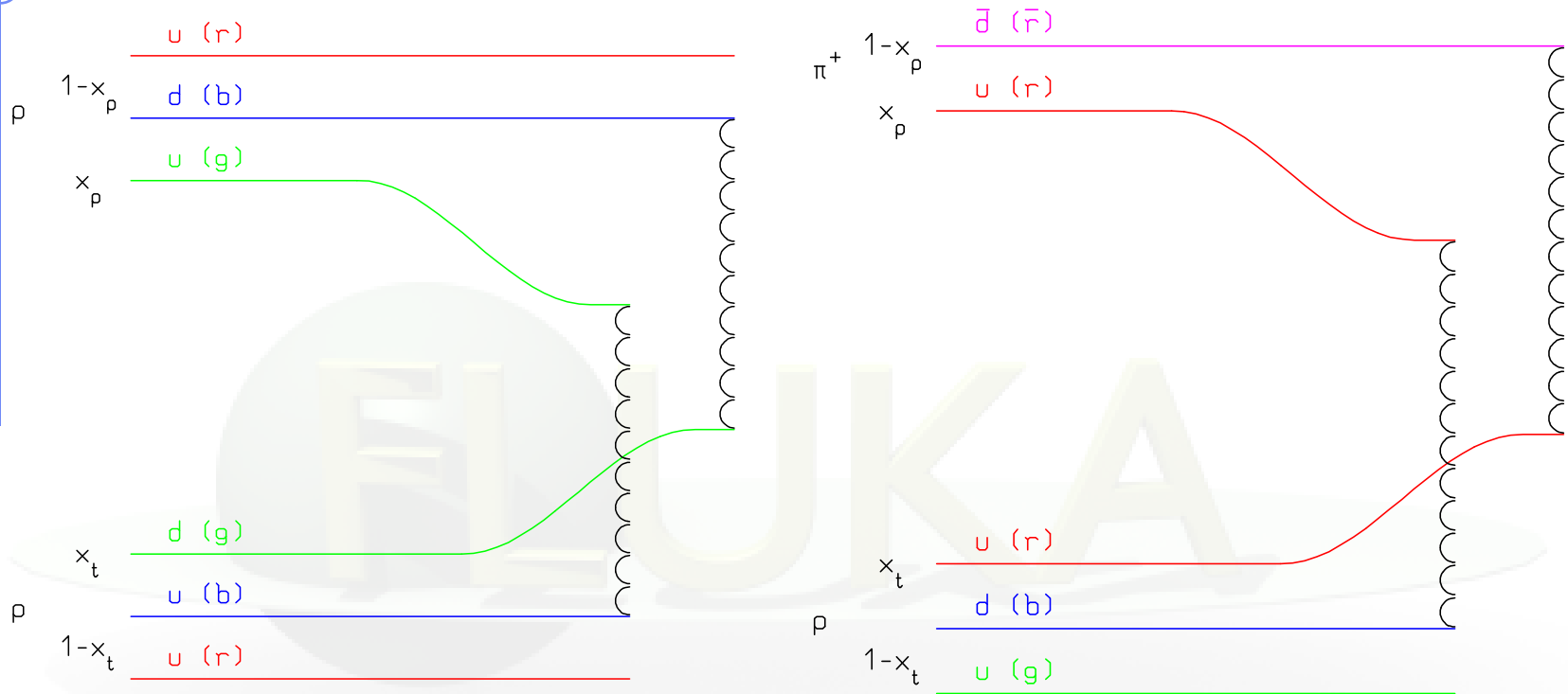
Resonance energies, widths, cross sections, branching ratios from data and conservation laws, whenever possible. Inferred from inclusive cross sections when needed



## Inelastic hN at high energies: (DPM, QGSM, ...)

- Problem: "soft" interactions → QCD perturbation theory cannot be applied.
- **Interacting strings** (quarks held together by the gluon-gluon interaction into the form of a string)
- Interactions treated in the Reggeon-Pomeron framework
- each of the two hadrons splits into **2 colored partons** → combination into **2 colourless chains** → **2 back-to-back jets**
- each jet is then **hadronized** into physical hadrons

# Hadron-hadron collisions: chain examples



Leading two-chain diagram in DPM for p-p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities

Leading two-chain diagram in DPM for  $\pi^+$ -p scattering. The color (red, blue, and green) and quark combination shown in the figure is just one of the allowed possibilities

# DPM and hadronization

## from DPM:

- Number of chains
- Chain composition
- Chain energies and momenta
- Diffractive events

Almost No Freedom

## Chain hadronization

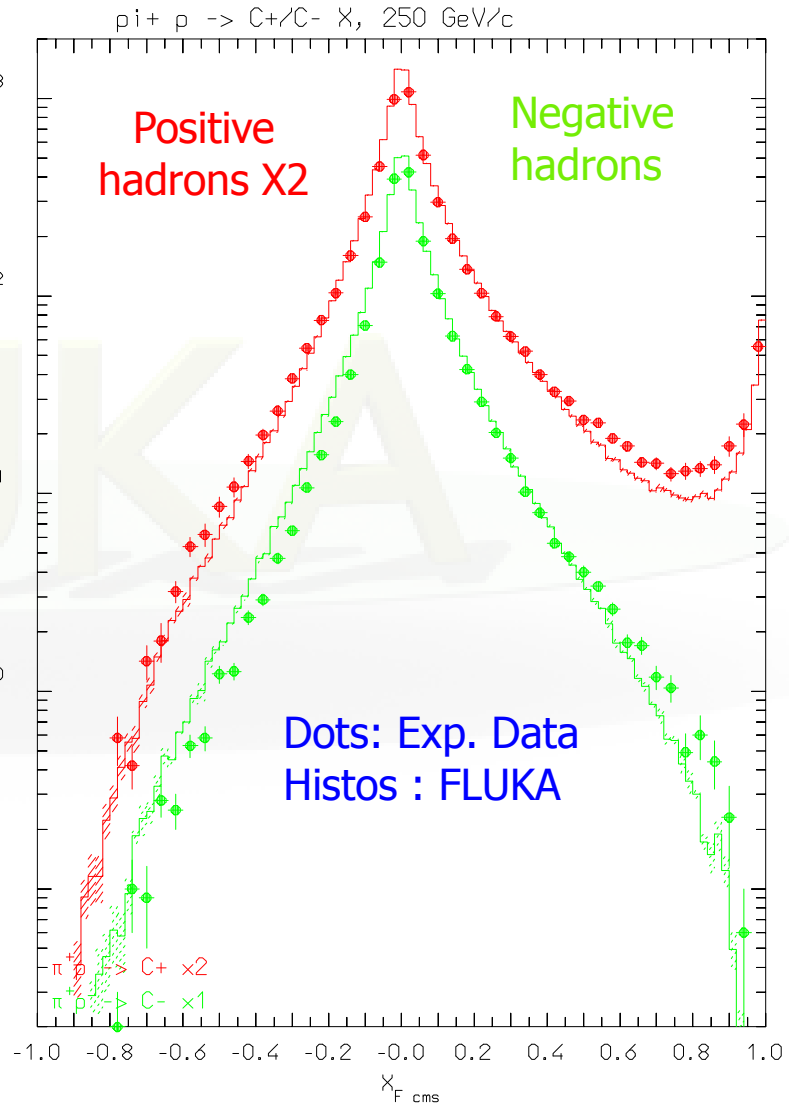
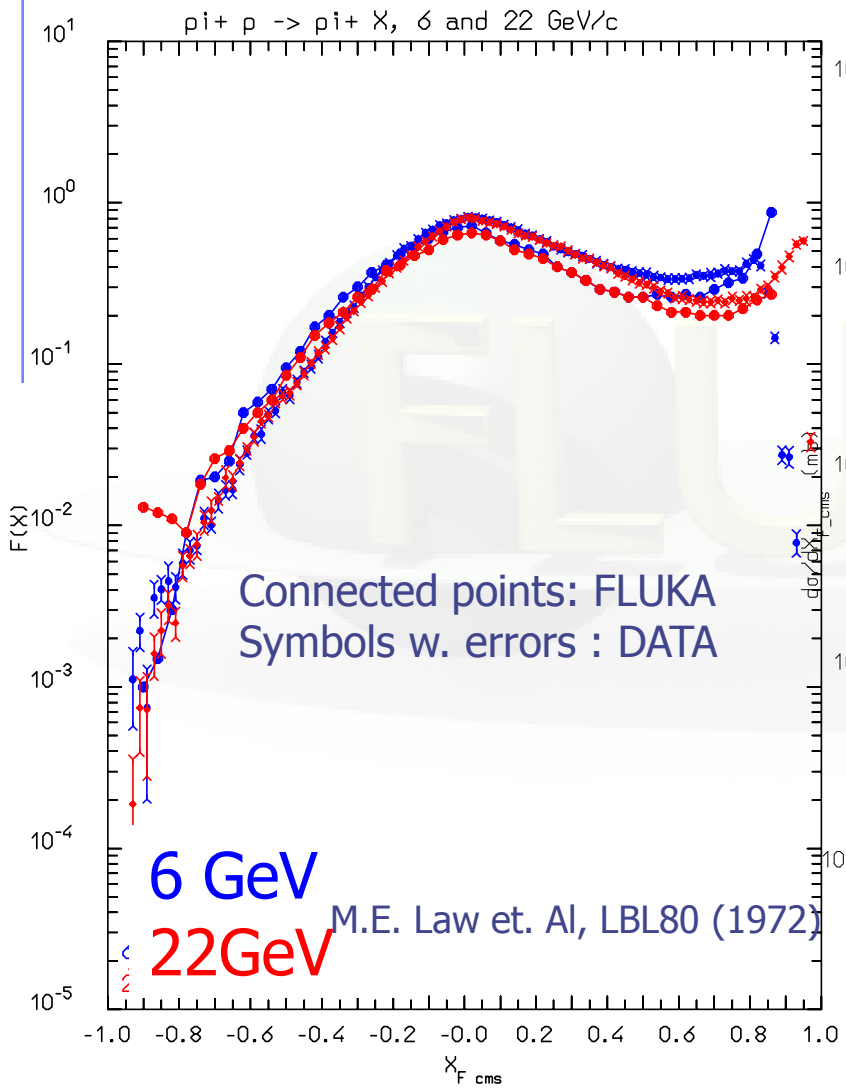
- Assumes chain universality
- Fragmentation functions from hard processes and  $e^+e^-$  scattering
- Transverse momentum from uncertainty considerations
- Mass effects at low energies

The same functions and (few) parameters for all reactions and energies

# Inelastic hN interactions: examples

$\pi^+ + p \rightarrow \pi^+ + X$  (6 & 22 GeV/c)

$\pi^+ + p \rightarrow Ch^+/Ch^- + X$  (250 GeV/c)





# Nuclear interactions in PEANUT:

Target nucleus description (density, Fermi motion, etc)



Glauber-Gribov cascade with formation zone



Generalized IntraNuclear cascade



Preequilibrium stage with current exciton configuration and excitation energy  
(all non-nucleons emitted/decayed + all nucleons below 30-100 MeV)



Evaporation/Fragmentation/Fission model



$\gamma$  deexcitation

$t$  (s)

$10^{-23}$

$10^{-22}$

$10^{-20}$

$10^{-16}$

# Nucleon Fermi Motion

- Fermi gas model: Nucleons = Non-interacting Constrained Fermions

**Momentum distribution**  $\propto \frac{dN}{dk} = \frac{|k|^2}{2\pi^2}$

for  $k$  up to a (local) Fermi momentum  $k_F(r)$  given by

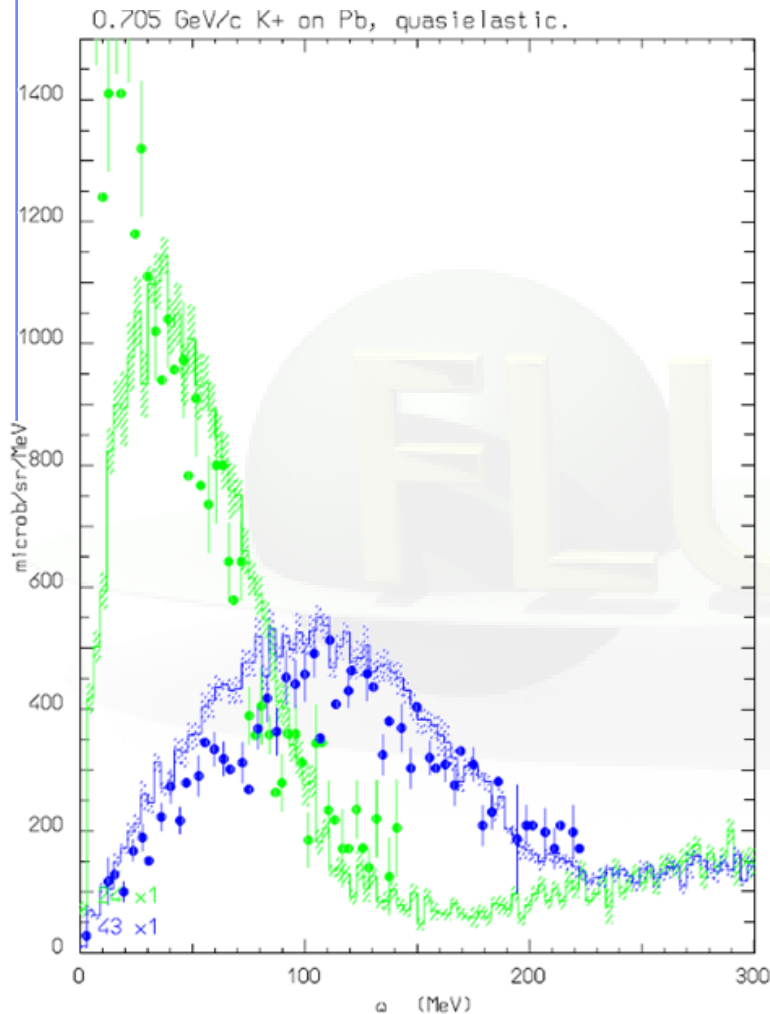
$$k_F(r) = \left[ 3\pi^2 \rho_N(r) \right]^{\frac{1}{3}}$$

The Fermi energy ( $k_F \approx 1.36$  fm,  $P_F \approx 260$  MeV/c,  $E_F \approx 35$  MeV, at nuclear max. density) is customarily used in building a self-consistent Nuclear Potential



Depth of the potential well  $\equiv$  **Fermi Energy** + **Nuclear Binding Energy**

# Positive kaons as a probe of Fermi motion



$$K^+ \quad K^0$$

No low mass  $S=1$  baryons  $\rightarrow$   
 weak  $K^+N$  interaction  
 only elastic and ch. exch. up to  
 $\approx 800 \text{ MeV}/c$

$(K^+, K^{*'})$  on Pb vs residual excitation, 705 MeV/c, at  $24^\circ$  and  $43^\circ$ .  
 Histo: FLUKA, dots: data (Phys Rev. C51, 669 (1995))

On free nucleon: recoil energy :  
 43 MeV at  $24^\circ$  , 117 MeV at  $43^\circ$ .

# (Generalized) IntraNuclear Cascade

- Primary and secondary particles moving in the nuclear medium
- Target nucleons motion and nuclear well according to the **Fermi gas model**
- Interaction probability  
 $\sigma_{\text{free}} + \text{Fermi motion} \times \rho(r) + \text{exceptions (ex. } \pi)$
- **Glauber cascade at higher energies**
- Classical trajectories (+) nuclear mean potential (**resonant for } \pi)**
- Curvature from nuclear potential  $\rightarrow$  **refraction and reflection**
- Interactions are incoherent and uncorrelated
- Interactions in projectile-target nucleon *CMS*  $\rightarrow$  Lorentz boosts
- **Multibody absorption for } \pi, \mu^-, K^-**
- **Quantum effects** (Pauli, formation zone, correlations...)
- **Exact conservation** of energy, momenta and all additive quantum numbers, including nuclear recoil

# hA at high energies: Glauber-Gribov cascade with formation zone

- Glauber cascade
  - Quantum mechanical method to compute Elastic, Quasi-elastic and Absorption hA cross sections from **Free hadron-nucleon scattering + nuclear ground state**
  - **Multiple Collision** expansion of the scattering amplitude
- Glauber-Gribov
  - **Field theory** formulation of Glauber model
  - Multiple collisions  $\leftrightarrow$  **Feynman diagrams**
  - High energies: exchange of one or more Pomerons with one or more target nucleons (a closed string exchange)
- Formation zone (=materialization time)

# Glauber Cascade

Quantum mechanical method to compute all relevant hadron-nucleus cross sections from hadron-nucleon scattering:  $S_{hN}(\vec{b}, s) = e^{i\chi_{hN}(\vec{b}, s)} = \eta_{hN}(\vec{b}, s)e^{2i\delta_{hN}(\vec{b}, s)}$

and nuclear ground state wave function  $\Psi_i$

**Total** 
$$\sigma_{hAT}(s) = 2 \int d^2\vec{b} \int d^3\vec{u} |\Psi_i(\vec{u})|^2 \left[ 1 - \prod_{j=1}^A \text{Re} S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right]$$

**Elastic** 
$$\sigma_{hAel}(s) = \int d^2\vec{b} \int d^3\vec{u} |\Psi_i(\vec{u})|^2 \left[ 1 - \prod_{j=1}^A S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right]^2$$

**Scattering** 
$$\sigma_{hA\Sigma f}(s) \equiv \sum_f \sigma_{hAfi}(s) = \int d^2\vec{b} \int d^3\vec{u} |\Psi_i(\vec{u})|^2 \left[ 1 - \prod_{j=1}^A S_{hN}(\vec{b} - \vec{r}_{j\perp}, s) \right]^2$$

**Absorption (particle prod.)**

Absorption probability over a given  $b$  and nucleon configuration

$$\sigma_{hAbs}(s) \equiv \sigma_{hAT}(s) - \sigma_{hA\Sigma f}(s)$$

$$= \int d^2\vec{b} \int d^3\vec{u} |\Psi_i(\vec{u})|^2 \left\{ 1 - \left[ \prod_{j=1}^A 1 - \left[ 1 - |S_{hN}(\vec{b} - \vec{r}_{j\perp}, s)|^2 \right] \right] \right\}$$

# Gribov interpretation of Glauber multiple collisions

*Therefore the absorption cross section is just the integral in the impact parameter plane of the probability of getting at least one non-elastic hadron-nucleon collision*

and the overall average number of collision is given by

$$\langle \nu \rangle = \frac{Z\sigma_{hpr} + N\sigma_{hnr}}{\sigma_{hAabs}}$$

- Glauber-Gribov model = Field theory formulation of Glauber model
- Multiple collision terms  $\Rightarrow$  Feynman graphs
- At high energies : exchange of one or more pomerons with one or more target nucleons
- In the Dual Parton Model language: (neglecting higher order diagrams):  
Interaction with  $n$  target nucleons  $\Rightarrow 2n$  chains
  - Two chains from projectile valence quarks + valence quarks of one target nucleon  $\Rightarrow$  valence-valence chains
  - $2(n-1)$  chains from sea quarks of the projectile + valence quarks of target nucleons  $\Rightarrow 2(n-1)$  sea-valence chains

# Formation zone

Naively: "materialization" time (originally proposed by Stodolski).  
Qualitative estimate:

In the frame where  $p_{\parallel} = 0$

$$\bar{t} = \Delta t \approx \frac{\hbar}{E_T} = \frac{\hbar}{\sqrt{p_T^2 + M^2}}$$

Particle proper time

$$\tau = \frac{M}{E_T} \bar{t} = \frac{\hbar M}{p_T^2 + M^2}$$

Going to the nucleus system

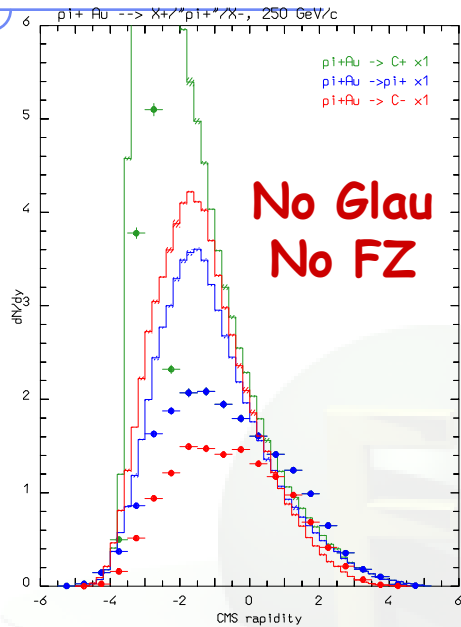
$$\Delta x_{for} \equiv \beta c \cdot t_{lab} \approx \frac{p_{lab}}{E_T} \bar{t} \approx \frac{p_{lab}}{M} \tau = k_{for} \frac{\hbar p_{lab}}{p_T^2 + M^2}$$

Condition for possible reinteraction inside a nucleus:

$$\Delta x_{for} \leq R_A \approx r_0 A^{\frac{1}{3}}$$

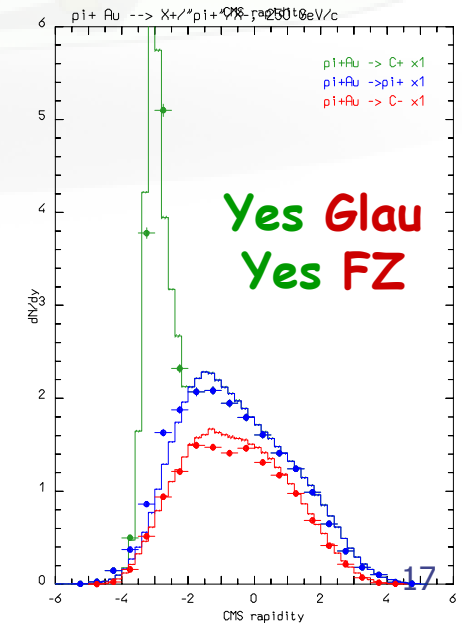
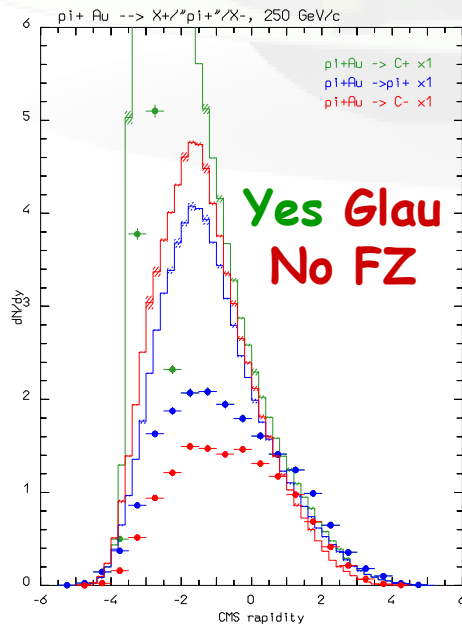
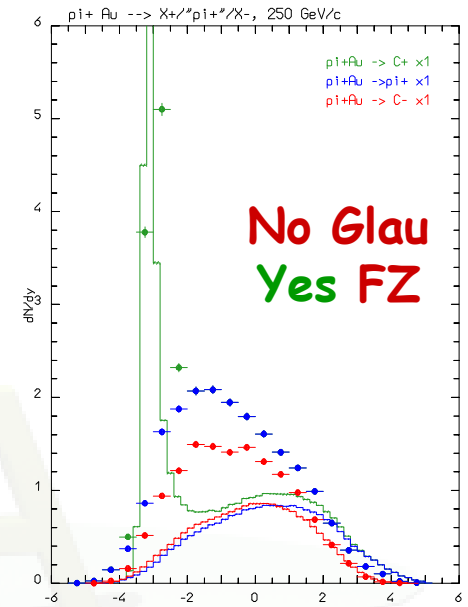


# Effect of Glauber and Formation Zone



Rapidity distribution of charged particles produced in 250 GeV  $\pi^+$  collisions on Gold

Points: exp. data (Agababyan et al., ZPC50, 361 (1991)).



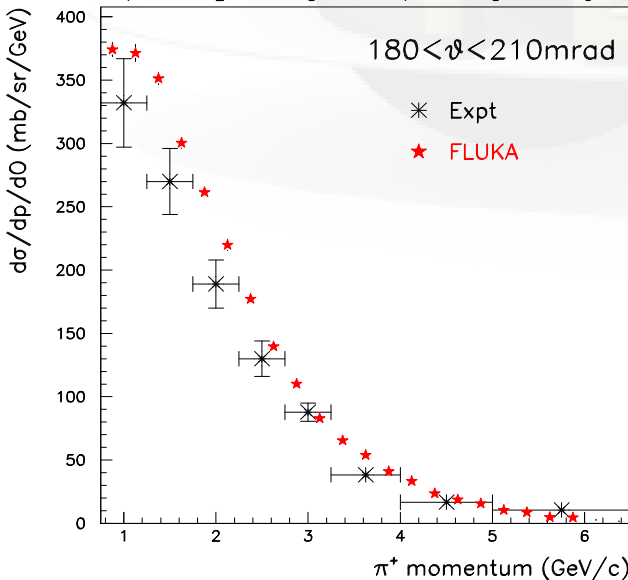
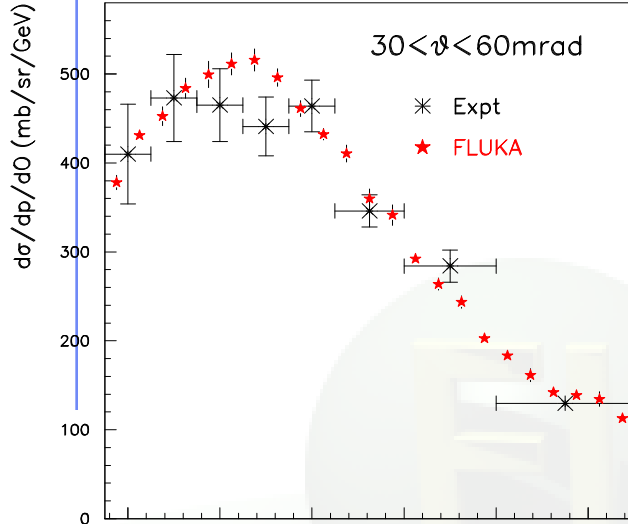
# WARNING

- PEANUT has been extended to cover the whole energy range in late 2006
- A less refined GINC model is available for projectile momenta above about 5 GeV/c, and was the only choice until recently
- However: the extended peanut is NOT yet the default, mainly because of some ambiguity in the definition of quasi-elastic cross sections.
- To activate PEANUT at all energies:

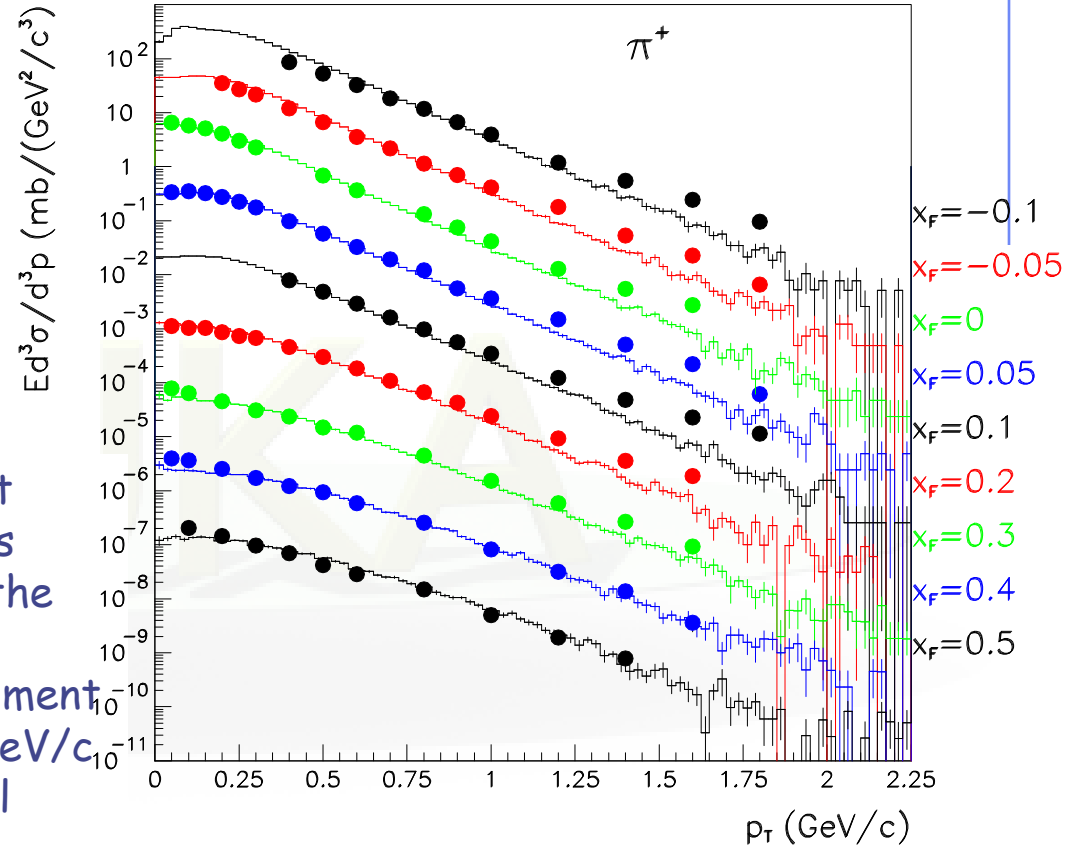
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# Nonelastic hA interactions at high energies: examples

Harp expt. , 12.9 GeV/c p on Al



NA49 expt. , 158 GeV/c p on C



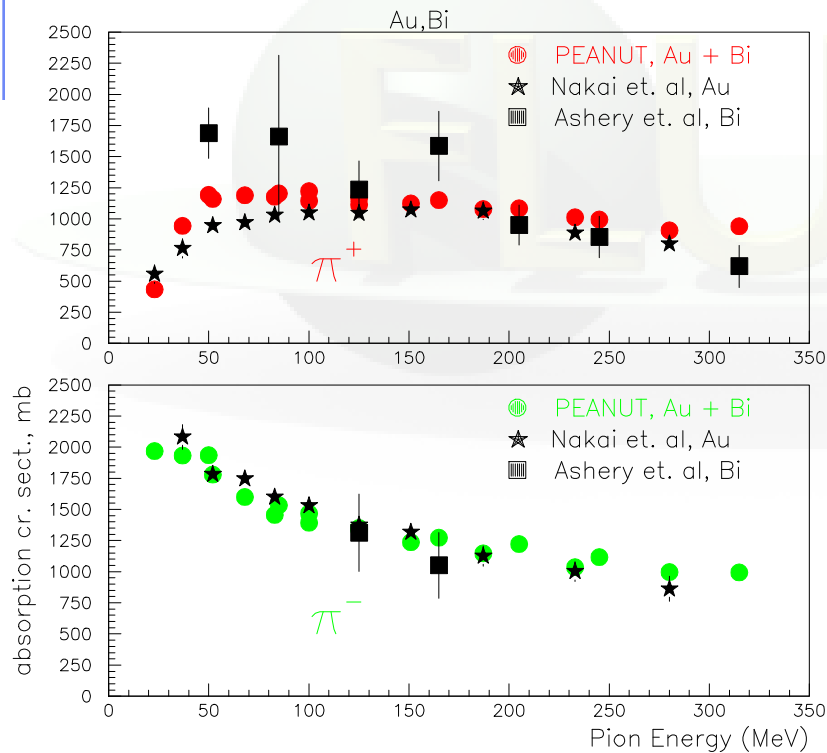
Recent results from the HARP experiment 12.9 GeV/c p on Al  $\pi^+$  production at different angles

Double differential  $\pi^+$  production for p C interactions at 158 GeV/c, as measured by NA49 (symbols) and predicted by FLUKA (histograms)



# Pion absorption

Pion absorption cross section on Gold and Bismuth in the  $\Delta$  resonance region (multibody absorption in PEANUT)

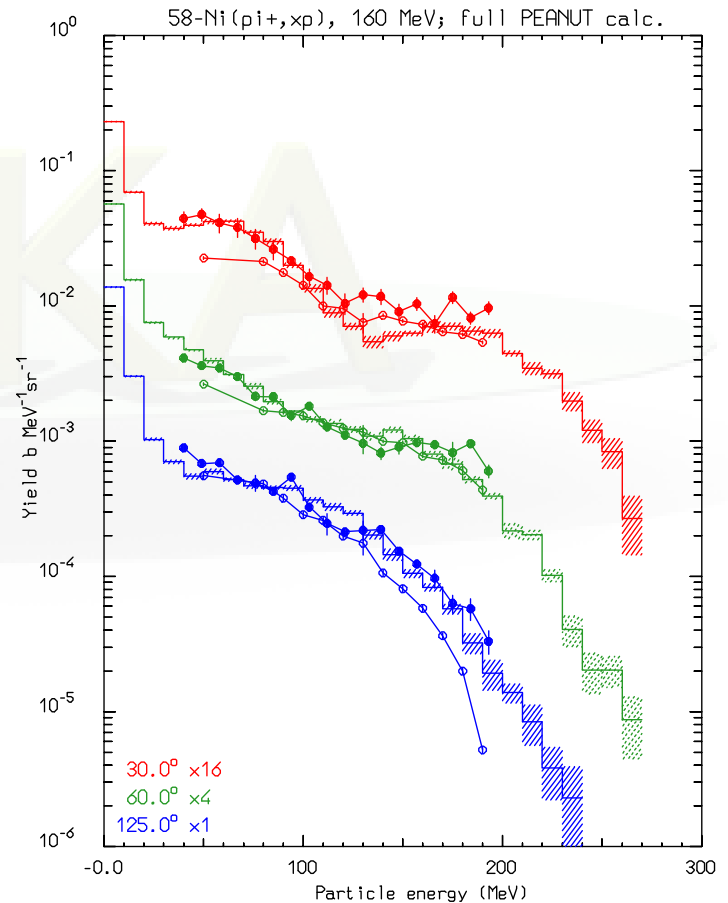


Emitted proton spectra at different angles, 160 MeV  $\pi^+$  on  $^{58}\text{Ni}$

Phys. Rev. C41,2215 (1990)

Phys. Rev. C24,211 (1981)

Proton spectra extend up to 300 MeV



# Preequilibrium emission

For  $E > \pi$  production threshold  $\rightarrow$  only (G)INC models  
At lower energies a variety of preequilibrium models

## Two leading approaches

The quantum-mechanical multistep model:  
Very good theoretical background  
Complex, difficulties for multiple emissions

The semiclassical exciton model  
Statistical assumptions  
Simple and fast  
Suitable for MC

### Statistical assumption:

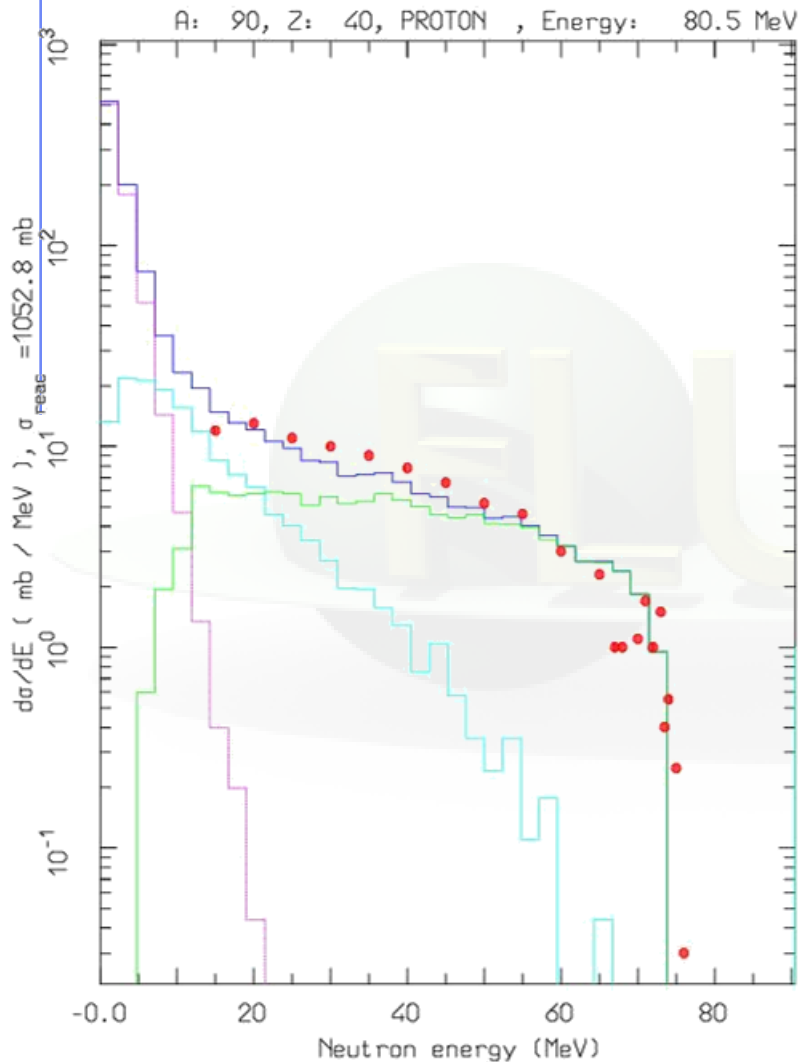
any partition of the excitation energy  $E^*$  among  $N$ ,  $N = N_h + N_p$ , excitons has the same probability to occur

Step: nucleon-nucleon collision with  $N_{n+1} = N_n + 2$  ("never come back approximation")

Chain end = equilibrium =  $N_n$  sufficiently high or excitation energy below threshold

*$N_1$  depends on the reaction type and cascade history*

# Thin target example

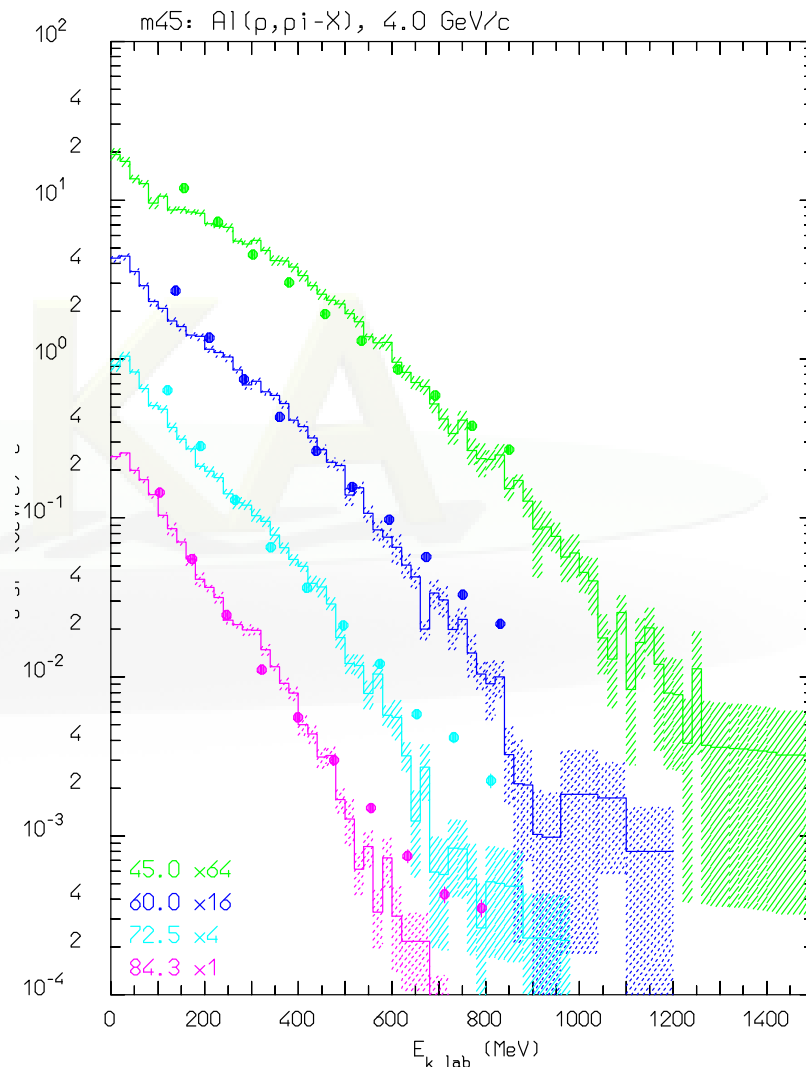
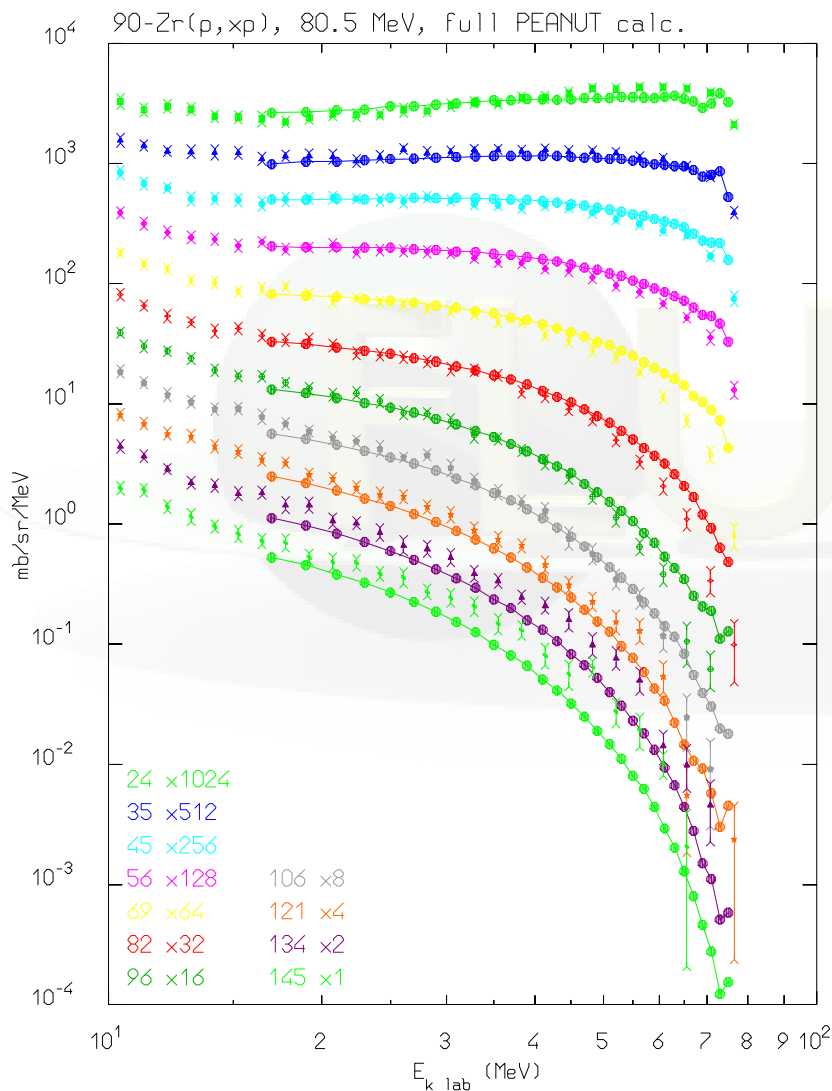


Angle-integrated  $^{90}\text{Zr}(p,xn)$  at 80.5 MeV

The various lines show the total, INC, preequilibrium and evaporation contributions

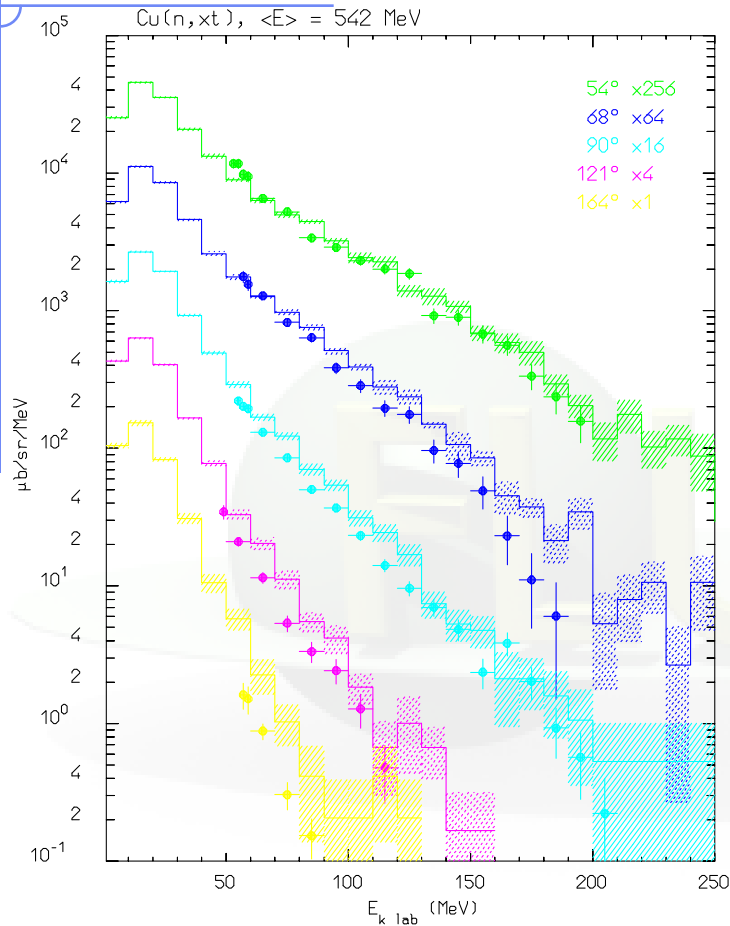
Experimental data from M. Trabandt et al., Phys. Rev. C39, 452 (1989)

# Thin target examples





# Coalescence



High energy light fragments are emitted through the coalescence mechanism: "put together" emitted nucleons that are near in phase space.

Example : double differential  $t$  production from 542 MeV neutrons on Copper

**Warning:** coalescence is OFF by default  
Can be important, ex for . residual nuclei.  
To activate it:

# Equilibrium particle emission (evaporation, fission and nuclear break-up)

From statistical considerations and the detailed balance principle, the probabilities for emitting a particle of mass  $m_j$ , spin  $S_j \hbar$  and energy  $E$ , or of fissioning are given by:

( $i, f$  for initial/final state,  $F_{\text{Fiss}}$  for fission saddle point)

Probability per unit time of emitting a particle  $j$  with energy  $E$

$$P_J = \frac{(2S_j + 1)m_j c}{\pi^2 \hbar^3} \int_{V_j}^{U_i - Q_j - \Delta_f} \frac{\rho_f(U_f)}{\rho_i(U_i)} \sigma_{\text{inv}}(E) E dE$$

Probability per unit time of fissioning

$$P_{F_{\text{Fiss}}} = \frac{1}{2\pi\hbar} \int_0^{U_i - B_{F_{\text{Fiss}}}} \frac{\rho_{F_{\text{Fiss}}}(U_i - B_{F_{\text{Fiss}}} - E)}{\rho_i(U_i)} dE$$

- $\rho$ 's: nuclear level densities
- $U$ 's: excitation energies
- $V_j$ 's: possible Coulomb barrier for emitting a particle type  $j$
- $B_{F_{\text{Fiss}}}$ : fission barrier

- $Q_j$ 's: reaction  $Q$  for emitting a particle type  $j$
- $\sigma_{\text{inv}}$ : cross section for the inverse process
- $\Delta$ 's: pairing energies

***Neutron emission is strongly favoured because of the lack of any barrier  
Heavy nuclei generally reach higher excitations because of more intense cascading***

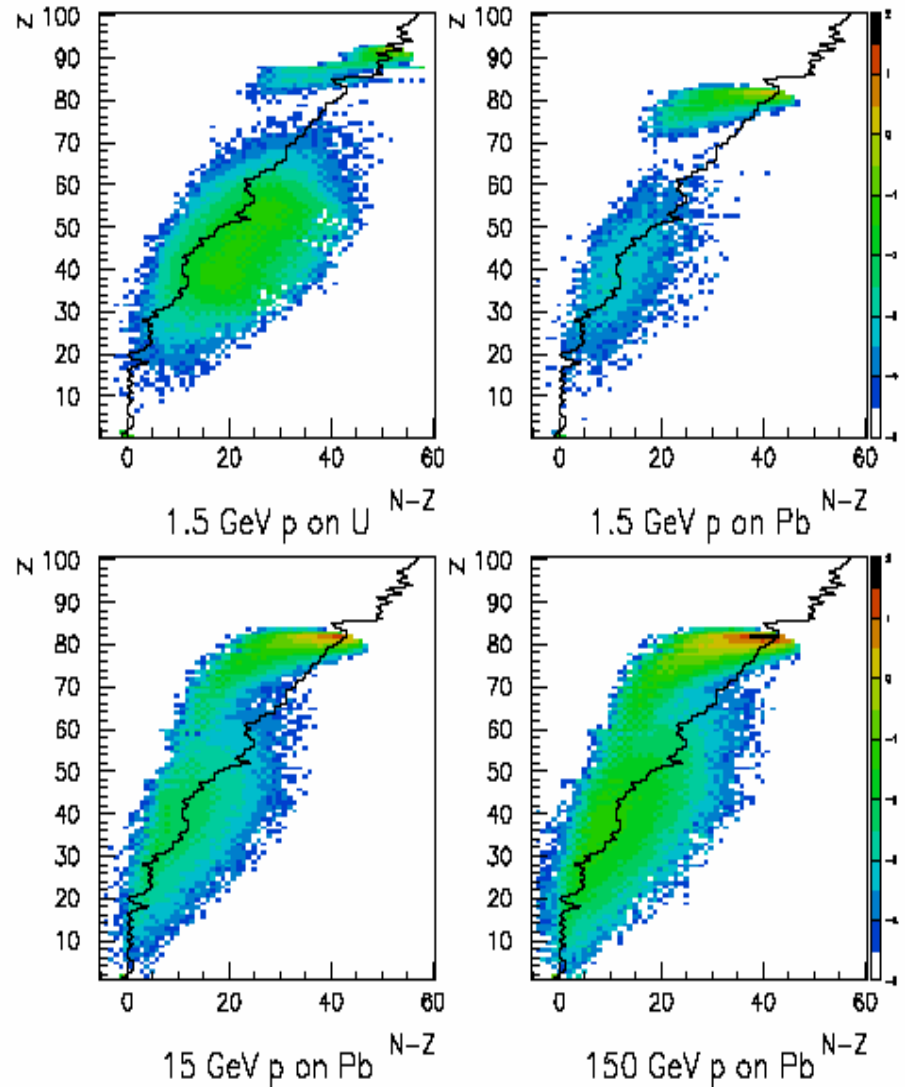
# Equilibrium particle emission

- Evaporation: Weisskopf-Ewing approach
  - ~600 possible emitted particles/states ( $A < 25$ ) with an extended evaporation/fragmentation formalism
  - Full level density formula with level density parameter  $A, Z$  and excitation dependent
  - Inverse cross section with proper sub-barrier
  - Analytic solution for the emission widths (neglecting the level density dependence on  $U$ , taken into account by rejection)
  - Emission energies from the width expression with no. approx.
- Fission: past, improved version of the Atchison algorithm, now
  - $\Gamma_{fis}$  based of first principles, full competition with evaporation
  - Improved mass and charge widths
  - Myers and Swiatecki fission barriers, with exc. en. Dependent level density enhancement at saddle point
- Fermi Break-up for  $A < 18$  nuclei
  - ~ 50000 combinations included with up to 6 ejectiles
  - $\gamma$  de-excitation: statistical + rotational + tabulated levels

# Residual Nuclei

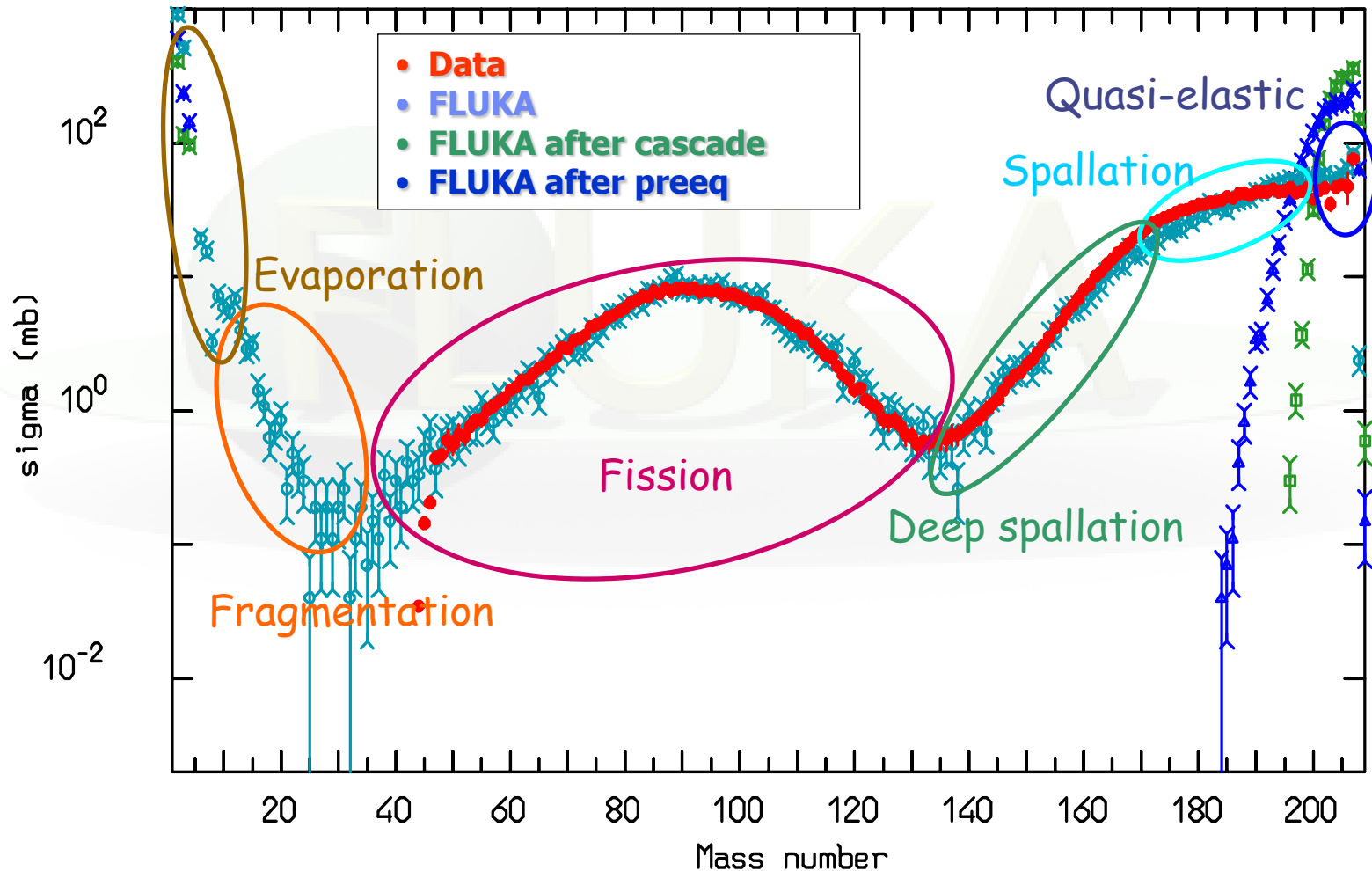
- The production of residuals is the result of the last step of the nuclear reaction, thus it is influenced by all the previous stages
- Residual mass distributions are very well reproduced
- Residuals near to the compound mass are usually well reproduced
- However, the production of specific isotopes may be influenced by additional problems which have little or no impact on the emitted particle spectra (Sensitive to details of evaporation, Nuclear structure effects, Lack of spin-parity dependent calculations in most MC models)

$\text{Log}_{10} N$  of residual nuclei



# Example of fission/evaporation

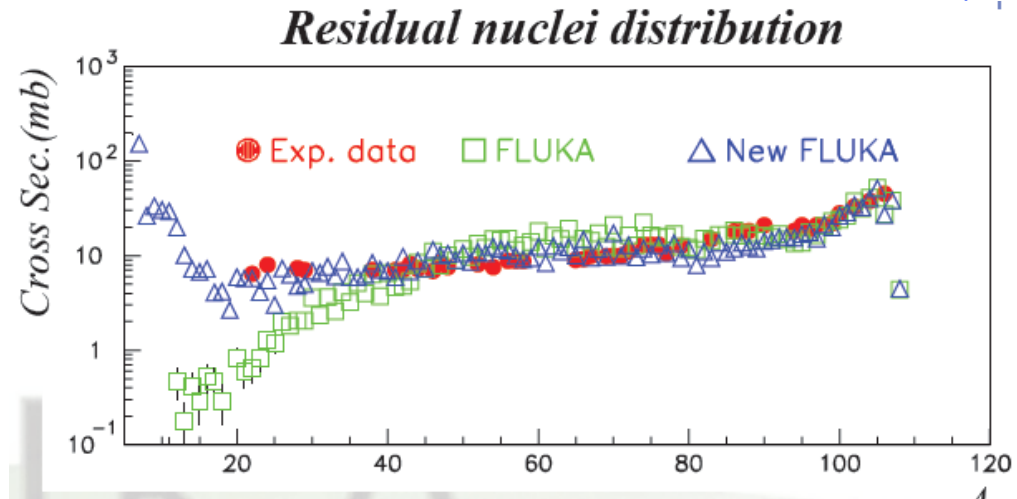
1 A GeV  $^{208}\text{Pb} + \text{p}$  reactions Nucl. Phys. A 686 (2001) 481-524



# Residual nuclei

Experimental and computed residual nuclei mass distribution for  $Ag(p,x)X$  at 300 GeV

Data from Phys. Rev. C19 2388 (1979) and Nucl. Phys. A543, 703 (1992)



The fragmentation model has much improved the FLUKA predictions

Also for A-A interactions

**Warning:** fragmentation is OFF by default, because it is a cpu-eater. It is **NECESSARY** to activate it for activation studies:

PHYSICS 3.

EVAPORAT

If fragmentation is on, switch on Heavy ion transport and interactions (see later)